Midterm 1, Math 123S, Classical and Modern Geometries

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Instructions: This is a 75 minute, closed book exam. No collaboration on this exam is allowed, and all answers should be written in the blue books provided. Write and sign the honor pledge on the outside of the first blue book, and make sure your name is on all of the blue books you turn in.

Express your answers in essay form so that all of your ideas are clearly presented. Partial credit will be given for partial solutions which are understandable. If you want to make a guess, clearly say so. Partial credit will be maximized if you accurately describe what you know and what you are not sure about. Each problem is worth 10 points. Good luck on the exam!

Problem 1. State and prove the Star Trek Lemma for a point on a circle. (You may assume that the center of the circle is inside the angle.)

Problem 2. Suppose triangle ABC is a right triangle with right angle at C. Prove that C is on the circle with diameter AB.

Problem 3. The Nine Point Circle Theorem: Given triangle ABC, prove that the following nine points lie on the same circle: the midpoints A',B',C' of the three sides, the bases D,E,F of the three altitudes, and the three midpoints A",B",C" of AH, BH, CH, where H is the orthocenter.

Problem 4. The Four Coin Problem: Suppose three congruent circles meet at a common point P and meet in pairs at the points A,B,C, as in the figure below. Show that the circumcircle of triangle ABC has the same radius as the original three circles.

Problem 5. Prove that all three angle bisectors of a triangle intersect at a common point.

Problem 6. Use Ceva's theorem to prove that the altitudes of a triangle ABC intersect at a common point. (Hint: Compute the lengths of the various segments using trigonometric functions in terms of the side lengths a,b,c and the angles A,B,C.