Math 108 Exam 3

Name: ______________________________

I have neither given nor received any unauthorized help on this exam and I have conducted myself within the guidelines of the Duke Community Standard.

Signature: ______________________________

Instructions: You may not use any notes, books, calculators or computers. A box is provided for your answer you must write your answer (and nothing else) in the box to receive full credit for the problem. Even if the correct answer appears somewhere else on the page, you will not receive full credit. Moreover, you must also **show the work** you did to arrive at the answer to receive full credit. If you are using a theorem to draw some conclusions quote the result. You have 75 minutes to answer all the questions.
Good Luck !

Question	Max Point	Score
1	20	
2	30	
3	20	
4	15	
5	15	
Total	100	

1. (a) (15 points) Using the **Laplace Transform** [1] show that for $m \neq n$:

$$\int_0^t sin(n(t-\tau))sin(m\tau)d\tau = \frac{m}{m^2-n^2}sin(nt) - \frac{n}{m^2-n^2}sin(mt)$$

Answer:

[1]The integral can be computed using integration by parts. I am not asking for this! You must use the Laplace transform in order to receive any credit.

(b) (5 points) Using part (a) conclude that.

$$\int_0^{2\pi} sin(m\tau)sin(n\tau)d\tau = 0 \tag{1}$$

You might need the identity $sin(u+v) = sin(u)cos(v)+cos(u)sin(v)$

Answer:

2. (a) (20 points) Compute the Fourier series of $f(x) = x^2$ for $-1 \leq x \leq 1$:

Answer:

(b) (10 points) Show that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. Justify your procedure.

Answer:

3. (20 points) Solve the Laplace equation:

$$\begin{cases} u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0 & 0 \le r \le 1 \quad 0 \le \theta < 2\pi \\ u(1,\theta) = f(\theta) & 0 \le \theta < 2\pi \end{cases}$$

where:

$$f(\theta) = \begin{cases} 1 & 0 \le \theta < \pi \\ 0 & \pi \le \theta < 2\pi \end{cases}$$

You are not required to derive the solution.

Answer:

4. (15 points) Determine whether or not the following boundary value problem has solutions:

$$\begin{cases} y'' + 4y = cos(x) \\ y(0) = 0 \\ y(\frac{3}{4}\pi) = 0 \end{cases}$$

Answer:

5. (15 points) The wave equation in two dimensions is:

$$u_{xx} + u_{yy} = a^{-2} u_{tt}$$

Assume that $u(x, y, t) = X(x)Y(y)T(t)$. Find the ordinary differential equations that X, Y and T have to satisfy in order for $u(x, y, t) = X(x)Y(y)T(t)$ to be a solution of the heat equation.

Answer: