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MATH 190S

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## Geometry of Gerrymandering

### **Introduction**

Before the Voting Rights Act of 1965, southern Democrats used to heavily suppress black representation in the U.S. House through drawing districts in such a way that diluted the black vote<sup>1</sup>. Political scientists call this practice “gerrymandering”. This practice has heavily evolved since that historic piece of legislation. The advent of computer programming – particularly GIS technology (geographic information system) – has enabled parties in power to perfectly maximize the number of districts they control in their states’ House delegations, but why do parties in power have so much influence? Why do they control the borders of the districts in their states? In the vast majority of states, the legislatures reapportion their states’ districts in each state after the decennial census. As a result, the majority party in the state legislature aims to draw borders that maximize the number of majority party members in the state’s delegation to the House. However, these parties must fall within certain parameters due to federal court and state court rulings on fairness in redistricting.

Firstly, historically minority-majority communities must be able to vote for a candidate of the majority ethnicity of that community. This condition leads to districts like Mississippi’s 2<sup>nd</sup> congressional district – a majority black district in a heavily Republican state. Many strong opponents of gerrymandering assume districts like MS-2 arise because the Republican state legislature wants to have safer seats for their three Republican House members from Mississippi

since African-American voters tend to heavily favor Democrats. In reality, the Republican Party could “crack” (a term discussed and explained in detail later) the black neighborhoods in Jackson – the urban area that MS-2 contains – and most likely find a way to control all four house districts in Mississippi. Instead, the Republican-controlled state legislature must have a district for the black-majority communities in Jackson in order to comply with state and federal courts’ interpretations of the Voting Rights Act of 1965.

Secondly, a district should respect existing political boundaries – such as the boundaries of counties and municipalities – as much as possible. Thirdly, a district must be as compact as possible. Finally, districts within a state should have approximately equal populations.<sup>3</sup>

Unfortunately, these parameters often conflict with each other. Existing political boundaries often *create* non-compact districts, and in order to create a district where historically majority-minority communities can elect someone of the majority ethnicity, sometimes state legislatures *have* to gerrymander. Due to these conflicting factors, one can quantify gerrymandering to a certain degree, but a human mind must incorporate other variables to assess the validity of a gerrymandered district.

## Section I: Background of Gerrymandering

### **Historical Background of Gerrymandering:**

Before one ponders how to measure gerrymandering, he must look back upon some political history in order to get context. In 1812, the Massachusetts state legislature redistricted the state senate in such a way to favor the Democratic-Republicans. Federalists – the opposition

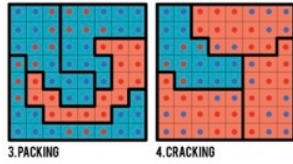


party in Massachusetts at the time – drew the following political cartoon in sympathetic newspapers in order to satirize the oddly shaped districts that the Democratic-Republicans created.

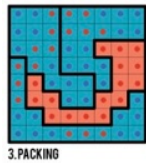
The districts that favored the Democratic-Republican Party form the shape of a serpentine creature. The cartoonist also cleverly gave the monster a dark color in order to imply wickedness of the Democratic-Republican Party as it encroaches in on the fairly drawn Federalists districts. Federalists began dubbing the creature a “gerrymander” – a portmanteau of the name of the amphibian salamander and the name of the Democratic-Republican Massachusetts Governor Elbridge Gerry.

### **Cracking & Packing:**

Gerrymandering comes in two different forms: cracking and packing. Cracking occurs when a voting bloc for the minority party concentrates in a certain area, and the majority party “cracks” this bloc and divides the population into multiple surrounding districts. If the legislature gerrymandered correctly, the surrounding districts will favor the majority party thereby suppressing the vote of the minority party

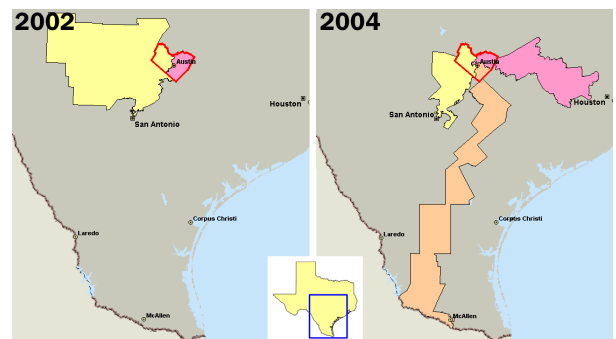


The image to the left contains the map of the hypothetical state of North Squarolina. In the center of North Squarolina lives twelve North Squarolinians who will most likely vote for the Blue Party in the next election for the Congruency Congress. The Red Party controls the state legislature and wants to maximize its number of districts in the North Squarolina congressional delegation, so it would have to “crack” the Blue voting bloc in central North Squarolina and divide that bloc in the three bordering pro-Red districts.



One election cycle passes, and the Blue Party has gained control of the legislature and can now redraw the boundaries of the districts. In order to maximize the number of *Blue* districts, the Blue Party can partake of packing and “pack” the likely Red voters into a single 100% Red-friendly district. The other three districts now have the spillover of that Red voting bloc, which constitute a minority of the voters in the three new pro-Blue districts.

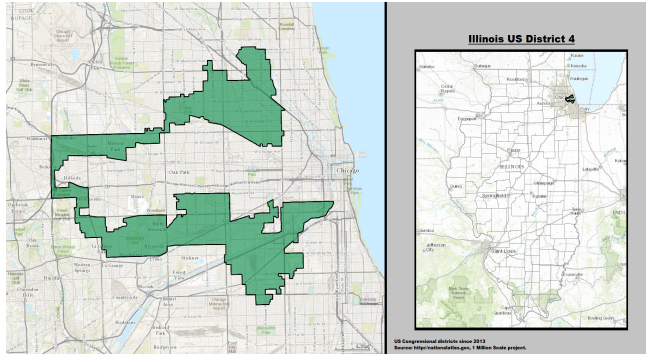
The map of southeastern Texas on the right demonstrates an example of cracking. After the 2000 Census, the Republican-controlled Texas state legislature wanted to diminish the power of the voters in Travis County – outlined red. Travis County contains Texas’s capital city, Austin, and typically favors Democrats (In 2016, its



residents voted 65.8% for the Democratic nominee, Secretary of State Hillary Clinton, and only 27.1% for current Republican President Donald Trump.<sup>3</sup>) The pink district in the 2002 map had a Democratic U.S. representative, but after the redistricting, the state legislature cracked Travis

County and diluted the power of the Democrats in the Austin area by the putting them in the Republican-friendly yellow and orange districts and the new pink district.

The infamous “earmuffs” district in Chicago in the image to the left demonstrates an example of packing. It packs together two majority Hispanic neighborhoods in Chicago – a predominantly Puerto Rican one in the north and a predominantly Mexican in the south<sup>5</sup>. The



thin strip in the western part of the district follows Interstate 294. The district contains the road itself but not the areas to the west and east of the road. Luis Gutiérrez, of Puerto Rican descent, has represented Illinois’s fourth district since 1993<sup>4</sup>. Without this instance of gerrymandering, Latinos in Chicago probably would not have a Latino representing them because the surrounding neighborhoods have either black majority, black plurality, white majority, or white plurality. The Supreme Court has dubbed this type of gerrymandering that allows a racial group to have a representative of their race in the House as “affirmative racial gerrymandering”<sup>6</sup>.

Ever since the Voting Rights Act of 1965, cracking has become less frequent relative to packing. Democrats in southern states would crack black neighborhoods into white Democrat districts in order to suppress black representation in the U.S. Congress. The Voting Rights Act dubbed this method “negative racial gerrymandering”<sup>7</sup> as opposed to affirmative racial gerrymandering, demonstrated in IL-4. This sort of gerrymandering has proven to give African-Americans more seats in the House in the past fifty-two years.

## **Section II: Different Methods Quantifying Gerrymandering:**

### **Geometric Compactness:**

If one asked a politician how to determine the severity of gerrymandering in a congressional district, the politician might say something to the effect of: “If it looks ‘weird’, then the district is gerrymandered!” This response satisfies my non-mathematically inclined side. After all, any reasonable person who understands the basics of gerrymandering can look at Illinois’s fourth congressional district – displayed in the previous section – and conclude that the state legislature has gerrymandered IL-4. On the other hand, that method of determining the level of gerrymandering does not satisfy mathematically inclined part of me. How can one assign a number to a district in order to quantify the level of gerrymandering in the district?

The Arizona Independent Redistricting Commission (AIRC) – a non-partisan body independent chosen by the Arizona state legislature -- has redrawn the U.S. House districts in Arizona since the 2000 U.S. Census. The AIRC uses the Polsby-Popper Test to measure the level of gerrymandering in a district<sup>8</sup>. To calculate the Polsby-Popper score of a district, one would conduct the following steps<sup>9</sup>:

1. Let  $P$  equal the perimeter of the district.
2. Record the total land area of the district.
3. Calculate the area of a circle with a circumference equal to  $P$ .
4. Let  $PP$  equal the Polsby-Popper score.
5.  $PP$  equals the ratio of the land area of the district to the area of the circle.

We can generalize the algorithm above into the follow equation:  $PP = \frac{4\pi A}{P^2}$ , where  $PP$  equals the Polsby-Popper score,  $A$  equals the land area of the district, and  $P$  equals the perimeter of the district.

A Polsby-Popper score must fall within the interval  $(0,1]$ . The interval excludes 0 because the area of a congressional district cannot equal 0. A score closer to 0 means a district has a low

level of compactness. A score of 1 would mean that the district has the shape of a circle and has maximum compactness. The AIRC uses the metric of compactness to determine the degree of gerrymandering because the formula punishes districts that have jagged protrusions along the border.

To internalize how the Polsby-Popper Test works, I will calculate the scores of the district in North Squarolina from the previous section. We will first be analyzing the packed Red district whereby each small square represents 1 square kilometer. We can calculate the area of the Red district just by counting the squares within it because the boundary is contiguous to the sides of squares:

$$A = 16 \text{ square kilometers}$$

Because the area one of the small squares equals 1 square kilometer, we can deduce that a side of a small square equals 1 kilometer. We can calculate the perimeter of the Red district by counting the number of the sides of squares along which the district boundary lies. The perimeter of the Red district equals 30 kilometers.

$$P = 30 \text{ kilometers}$$

Now, we can substitute A for 16 and P for 30 in the formula for Polsby-Popper score.

$$PP = \frac{4\pi \cdot 16}{30^2} = 0.223$$

After calculating for the Red district, I have gone on and applied the Polsby-Popper Test for the other three districts. From the data gained, we can conclude that the top left Blue and top right Blue districts have the lowest degrees of gerrymandering because they have the highest Polsby-Popper scores. The bottom blue district has the highest degree of gerrymandering, and the Red district has the second highest degree of gerrymandering.

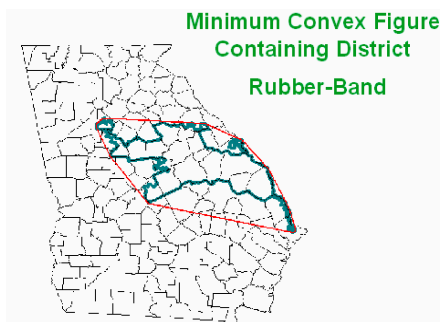
District	Area	Perimeter	Polsby-Popper Score
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Red	16	30	0.223
Top Left Blue	16	22	0.415
Top Right Blue	16	22	0.415
Bottom Blue	16	33	0.185

This method is relatively straightforward with regards to our utopia of North Squarolina, but the Polsby-Popper Test gets more problematic when we apply it to actual House districts in the United States. Contiguous squares do not comprise the area and outline of states thereby making the calculation by hand quite difficult. State government do not publish data on the perimeters of their states and the districts within these states, so I cannot calculate the Polsby-Popper values by hand. Fortunately, a man named Christopher Ingraham conducted an exhaustive analysis of the Polsby-Popper scores for each district in the United States<sup>10</sup>.

He first downloaded shapefiles for each 2013 U.S. house district from the U.S. Census.

Not only do the shapefiles for the districts contain the coordinate locations of the vertices in the



boundaries, but they also change the boundaries of districts with irregular coastlines by using the rubber band method whereby a convex figure with minimum area circumscribes the district as if one were wrapping a rubber band around it. The map of Georgia to the right demonstrates this method with respect to a now defunct

congressional district. Though the district no longer exists, it still accurately demonstrates the rubber band method.

The shapefiles' use of the rubber band method benefitted Ingraham because coastlines unfairly increase the perimeter of the state because coastlines have many small inlets and outlets.



Redistricting entities in highly coastal states – such as California and Florida – have a difficult time creating geometrical compact districts because the states themselves do not possess geometric compactness due to the irregular coastlines. On the other hand, a landlocked and relatively compact state like Iowa has a much easier time drawing compact districts because Iowa has no irregular coastline. Though the rubber band method of outlining coastlines does not *completely* mitigate the effects of irregular coastlines, it mitigates the effect enough to at least fairly compare gerrymandering in coastal states like California with that in landlocked state like Iowa.

Even if a state does not border a large body of water, it still may have a disadvantage with regards to drawing compact districts because it may have irregular borders with the states surrounding it. In the map of West Virginia to the right, one can see that Mountain State has jagged, irregular borders with Ohio, Pennsylvania, Maryland, and Virginia. The West Virginia state legislature cannot control the boundary of their state, so the legislators have another disadvantage with drawing compact district just as Florida and California do. Unfortunately, not even can the rubber band correct for these irregular interstate borders because if one were to wrap a metaphorical rubber band around West Virginia, it would pass through the states bordering West Virginia. The beauty of the rubber band method along coastlines lies in the fact that wrapping a rubber band around a coastline does not affect the geometry of another state. It only affects the geometry of the body of water that the coastline borders – which does not matter for redistricting.



Once Ingraham retrieved these shapefiles, which correct for irregular coastlines, he inputted them into Quantum Geographic Information System (QGIS) -- a complicated

programming interface that can accept shapefiles and can calculate the perimeters of each House district. I wish that I had the programming background to operate QGIS myself – as Ingraham did – or the time to learn how to use it, but unfortunately, I have a sufficient amount of neither. After calculating the perimeters and the areas of each district, he calculated the Polsby-Popper scores for all of them.

Ingraham did not calculate the Polsby-Popper scores for states with one district (Alaska, Delaware, Montana, North Dakota, South Dakota, Vermont, and Wyoming) because the state governments in these states do not need to redraw the district at all. The state boundaries already did the work for them! Unfortunately, Ingraham did not calculate for the scores for states with two districts either (Hawaii, Idaho, Maine, New Hampshire, and Rhode Island) because he claims that the redistricting entities in these states are at the behest of the state boundaries. They can only draw a jagged line through the center of these states. Though I agree with Ingraham's decision to exclude them, he does not extend this logic far enough. Ingraham's rationale implies that interstate boundaries do not affect the borders of districts in states with at least three U.S. House representatives, but I would argue the contrary because the borders of a state still have a huge impact on any district that lies on the border of any state.

### **Section III: Analysis of Ingraham's Calculations**

After analyzing the Ingraham data, I compiled the top 10 *most* and top 10 *least* gerrymandered districts.

#### **Top 10 Most Gerrymandered Congressional Districts**

1. NC-12
2. MD-3
3. FL-5
4. PA-7
5. NC-1
6. TX-33

7. NC-4
8. IL-4
9. TX-35
10. LA-2

Top 10 Least Gerrymandered Congressional District

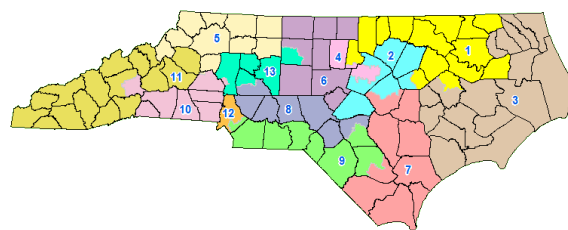
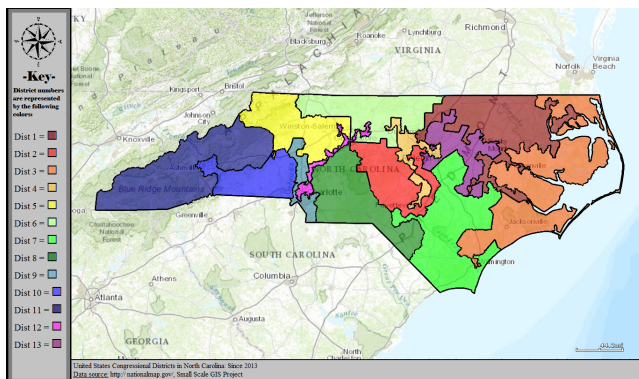
1. IN-1
2. NV-1
3. NV-2
4. IN-3
5. NY-15
6. TX-16
7. MN-4
8. MI-6
9. AZ-5
10. NV-4

When it comes to gerrymandering with regards to entire states, the following list ranks the 38 analyzed states by median Polsby-Popper score from least to greatest with the actual score in parentheses to the right of the name of the state

1. Maryland (0.084)
2. **North Carolina (0.107)**
3. Louisiana (0.127)
4. Pennsylvania (0.130)
5. West Virginia (0.139)
6. Kentucky (0.144)
7. Ohio (0.146)
8. Illinois (0.158)
9. Virginia (0.163)
10. Texas (0.179)
11. Alabama (0.182)
12. Arkansas (0.190)
13. New Jersey (0.198)
14. South Carolina (0.207)
15. Massachusetts (0.213)
16. Tennessee (0.215)
17. California (0.220)
18. Colorado (0.225)
19. Missouri (0.229)
20. Washington (0.229)
21. Connecticut (0.231)
22. Mississippi (0.249)

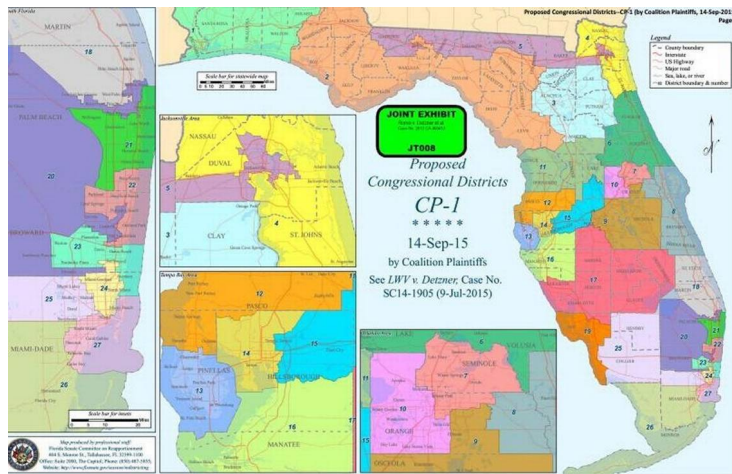
23. Oklahoma (0.250)
24. Michigan (0.254)
25. Wisconsin (0.256)
26. Utah (0.256)
27. Oregon (0.260)
28. Georgia (0.268)
29. Minnesota (0.279)
- 30. Florida (0.307)**
31. Arizona (0.319)
32. Nebraska (0.329)
33. New York (0.330)
34. Iowa (0.348)
35. New Mexico (0.365)
36. Kansas (0.392)
37. Indiana (0.461)
38. Nevada (0.528)

In the lists above, North Carolina, Florida, and the districts within those states have bolded font since the boundaries of the districts within the Sunshine State and Tar Heel State have changed since the constitutionally required redistricting after the 2010 U.S. Census for the 2012 elections<sup>11,12</sup>. In North Carolina, Chief Judge Roger Gregory of the U.S. Court of Appeals for the Fourth Circuit ordered the Tar Heel State to redraw its districts in time for the 2016 elections because he deemed that the state legislature had racially gerrymandered North Carolina.



The map on the left shows the districts before the redrawing for the 2016 elections, and the map on the right shows the districts after the redrawing.

As for Florida, the Florida Supreme Court ruled that the state legislature had to redraw the boundaries of Florida's congressional districts because of excessive gerrymandering. The boundaries of every congressional district in the Sunshine State changed except for FL-18 and



FL-19. The map to the left shows the congressional districts between January 3, 2013 and January 3, 2017, and the map to the right shows the proposal to the Florida Supreme Court for the redistricting for the 2016 elections. The Florida Supreme Court deemed this proposal as having sufficiently fair districting.

These court cases may seem inconsequential to the mathematics of gerrymandering, but these new districts greatly affect the analysis of the Polsby-Popper scores for each district. Unfortunately, Ingraham has only analyzed the shapefiles for the districts drawn for the 2012 elections. Therefore, Ingraham's data for all congressional districts in Florida besides FL-18 and FL-19 and all congressional districts in North Carolina is inaccurate, so I want to exclude these data points from the data set because they are moot. Unfortunately, I do not have the knowledge of GIS in order to calculate the perimeters of the new districts in Florida and North Carolina, so I must rely on the calculations that Ingraham made for the districts that took effect on January 3, 2013. The following list ranks the bottom ten least compact congressional districts after

excluding all districts in North Carolina and all districts in Florida besides FL-18 and FL-19.

This list does not imply that the redistricting in the Sunshine and Tar Heel States made every district relatively compact. By eyeballing the new maps, somebody could reasonably conjecture that gerrymandering still has prevalence in these two states, and any of these districts could easily rank in the least compact congressional districts if the data was sufficient to provide the new Polsby-Popper scores. However, nobody can make these conclusions definitively without the data. One can only look at the districts with accurate and current Polsby-Popper scores and keep in mind that the data does not include Florida or North Carolina -- two states with a high number of congressional districts.

#### 10 Least Compact Congressional Districts

1. MD-3
2. PA-7
3. TX-33
4. IL-4
5. TX-35
6. LA-2
7. OH-3
8. MD-2
9. MD-6
10. MA-7

## Conclusion

Unfortunately, I do not have enough space in this paper to delve into other aspects of gerrymandering. Additionally, I did not have enough time to go fully in detail with other ways one can apply statistics and geometry to the analysis of gerrymandering due to the amount of time necessary for compiling all the data required for these analyses. I only went in-depth on the quantification of compactness, but I did not delve into the spread in the populations of district within states, the hypothesis testing with regards to fair racial representation in the House, or the quantification of how much a district boundary respects the existing political boundaries in a state. As long as Dr. Bray permits, I would like to further delve into these topics in at least one of my two future papers in *The Mathematics of the Universe*.

Nevertheless, with regards of the analysis I *did* conduct in this paper, I would question the AIRC's emphasis on the Polsby-Popper Test. Arizona serves as a policy precedent for other states to consider with respect to developing bipartisan and non-partisan redistricting committees, and I fear other states may too closely replicate the Arizona model. The Polsby-Popper Test works well for states with the following qualities to some degree:

1. The state has little to no coastline.
2. Its borders with other states, Mexico, or Canada, are not excessively jagged  
(Maryland and West Virginia are good examples of states that do *not* satisfy this condition)
3. The state is a relatively compact geometric shape itself.
4. The state has a large number of districts that do not lie on the state's border.

Texas provides a good example of this condition.

Redistricting entities must weigh these four factors depending on the state for which they are drawing new congressional districts.

Vehement opponents of gerrymandering need to consider the positive impacts of gerrymandering. Affirmative racial gerrymandering creates districts for minority groups so that they can have representation in the House. Much of the opposition against gerrymandering implies that all Republican representatives are the same and all Democratic representatives are the same. A Democrat representative in Boston, Massachusetts, is very culturally different from a Democrat representative from Jackson, Mississippi. Going forward, I would like to focus on determining other methods that quantify other factors for gerrymandering because compactness is not the only influence. No metric can stand alone in quantifying gerrymandering. Redistricting entities need to take multiple measures into account.

#### **Footnotes:**

1. [https://www.jstor.org/stable/796320?seq=1#page\\_scan\\_tab\\_contents](https://www.jstor.org/stable/796320?seq=1#page_scan_tab_contents)
2. <http://www.esri.com/industries/elections/redistricting>
3. <http://traviselectionresults.com/enr/results/display.do;jsessionid=09e4d06a493c7c102e33fcc5254d?criteria.electionId=201611&electionId=201611&tabType=C&formSubmitted=1>
4. <https://gutierrez.house.gov/>
5. <http://www.latina.com/celebrity/latino-stars-chicago-home>
6. <https://litigation-essentials.lexisnexis.com/webcd/app?action=DocumentDisplay&crawlid=1&srctype=smi>



<http://www.berkeley.edu/cgi/viewcontent.cgi?article=1810&context=facpubs&srcid=3B15&doctype=cite&docid=26+Cumb.+L.+Rev.+313&key=5c62a7998fbcf187f55f14357f22f082>

7. <http://scholarship.law.berkeley.edu/cgi/viewcontent.cgi?article=1810&context=facpubs>
8. <http://azredistricting.org/>
9. [http://www.math.cornell.edu/~ismythe/Lec\\_15\\_web.pdf](http://www.math.cornell.edu/~ismythe/Lec_15_web.pdf)
10. [https://www.washingtonpost.com/news/wonk/wp/2014/05/15/americas-most-gerrymandered-congressional-districts/?utm\\_term=.ecbb3cb57d87](https://www.washingtonpost.com/news/wonk/wp/2014/05/15/americas-most-gerrymandered-congressional-districts/?utm_term=.ecbb3cb57d87)
11. <http://www.charlotteobserver.com/news/politics-government/article58760423.html>
- 12.

### **Images:**

1. Gerrymander cartoon:

[https://en.wikipedia.org/wiki/Gerrymandering\\_in\\_the\\_United\\_States#/media/File:The\\_Gerry-Mander\\_Edit.png](https://en.wikipedia.org/wiki/Gerrymandering_in_the_United_States#/media/File:The_Gerry-Mander_Edit.png)

2. Cracking and packing map:

<https://cdn.azavea.com/com.redistrictingthenation/images/national/glossary-packing-cracking-01.png>

3. Map of Travis County, Texas:

[https://en.wikipedia.org/wiki/Gerrymandering\\_in\\_the\\_United\\_States#/media/File:TravisCountyDistricts.png](https://en.wikipedia.org/wiki/Gerrymandering_in_the_United_States#/media/File:TravisCountyDistricts.png)

4. Map of Illinois's 4<sup>th</sup> congressional district:

[https://upload.wikimedia.org/wikipedia/commons/thumb/d/d2/Illinois\\_US\\_Congressional\\_District\\_4\\_%28since\\_2013%29.tif/lossless-page1-400px-Illinois\\_US\\_Congressional\\_District\\_4\\_%28since\\_2013%29.tif.png](https://upload.wikimedia.org/wikipedia/commons/thumb/d/d2/Illinois_US_Congressional_District_4_%28since_2013%29.tif/lossless-page1-400px-Illinois_US_Congressional_District_4_%28since_2013%29.tif.png)

5. Rubber Band around district in Georgia:

<http://www.ncsl.org/documents/summit/hofeller.pdf>

6. Map of West Virginia: <https://i.infopls.com/images/mwestvirginia.gif>

7. North Carolina congressional map since January 3, 2017:

<https://commons.wikimedia.org/wiki/File:NorthCarolina2016USHouseDistricts.png>

8. North Carolina congressional map from January 3, 2013 to January 3, 2017:

[https://commons.wikimedia.org/wiki/File:United\\_States\\_Congressional\\_Districts\\_in\\_North\\_Carolina\\_since\\_2013.tif](https://commons.wikimedia.org/wiki/File:United_States_Congressional_Districts_in_North_Carolina_since_2013.tif)

9. Florida congressional map since January 3, 2017:

[https://commons.wikimedia.org/wiki/File:Florida\\_congressional\\_districts.png](https://commons.wikimedia.org/wiki/File:Florida_congressional_districts.png)

10. Florida congressional map between January 3, 2013 to January 3, 2017:

[https://commons.wikimedia.org/wiki/File:Florida\\_Congressional\\_Districts\\_113th\\_Congress.tif](https://commons.wikimedia.org/wiki/File:Florida_Congressional_Districts_113th_Congress.tif)