Duke University

The Prisoner's Dilemma

Shiqi Wang

MATH 190S.01M-mathematics of the Universe

Professor Hubert Bray

July 19, 2017

Introduction

When a market has only two competitors, the business form is called the Oligopoly. The two companies often have similar products, customers and business chain. So the aspect that really distinguishes the two companies is their company strategy. The study of the strategy is now called the Game Theory. Officially, Game Theory is "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers." It is mainly used in economics, political science, and psychology, as well as logic, computer science and biology (Myerson 1991). In the Game Theory there is an interesting situation which the common senses are not right, which is called the Prison's Dilemma.

Dominate Strategy

A strategy is dominant if, regardless of what any other players do, the strategy earns a player a larger payoff than any other. The Prisoner's Dilemma is based on the dominant strategy. To make a better understanding, here is an example:

Company A and Company B are the only two companies in the apple market which sell similar qualities of apples. Their market officers make a assumption on the sales of the apples based on whether the companies will advertise (Figure 1).

A \ B	Advertise (tons)	Not Advertise (tons)
Advertise (tons)	500, 500	600, 200
Not Advertise (tons)	200, 600	400, 400

Figure 1 The sales of apples for two companies

From the information above, when B advertises, A had better advertise too so it can sell 500 than 200; similarly, when B doesn't advertise, A should advertise so it can sell more. Thus, whatever B's action is, A should advertise. So advertise is A's dominant strategy.

However, if A has some advertising problems and caused the sales to decrease by one half (Figure 2), the strategy will change.

A \ B	Advertise (tons)	Not Advertise (tons)
Advertise (tons)	250, 500	300, 200
Not Advertise (tons)	200, 600	400, 400

Figure 2 Sales of the apples after the problems

From the figure, when B doesn't advertise, A should not too. Then there isn't a strategy for A to use. Thus, A has no dominant strategy in the business.

Nash Equilibrium

In game theory, the Nash equilibrium is a solution concept of a non-cooperative game involving two or more players in which each player

is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his own strategy. (Osborne, Ariel 1994) In other ways, Nash equilibrium is the combination of the dominant strategy. Look at the example below:

Considering two hamburger shops A and B. They are the only two burger shops in the district. Now the shop has to decide whether to open the shop at north part or south. The assumed sales are below. (Figure 3)

A / B	North	South
North	55 / 55	60 / 70
South	50 / 40	25 / 50

Figure 3 The sales of burgers

Combining the previous information, the dominant strategy of A is to open at the North while of B is to open at the South. And after combine the information together, it is clear that… (Figure 4)

A / B	North	South
North	55 / 55	60 / 70
South	50 / 40	25 / 50

Figure 4 The Nash Equilibrium of the sales (PURPLE)

The blue part is the dominant strategy of A; the red part is the dominant part of B. And their intersection, the purple part, is called the Nash Equilibrium. The Nash equilibrium was named after John Forbes Nash, Jr.

A version of the Nash equilibrium concept was first known to be used in 1838 by Antoine Augustin Cournot in his theory of oligopoly. (Cournot 1838) And according to Nash, “an equilibrium point is an n-tuple such that each player's mixed strategy maximizes his payoff if the strategies of the others are held fixed. Thus each player's strategy is optimal against those of the others. ” The purple block is where the profits of A and B are maximized. And any shop, if changes strategy cannot get full profits.

Prisoner’s Dilemma

As Nash Equilibrium represents the max profits of the oligopoly, there will be some special situation where both sides cannot get their ideal profits. An example may show the situation most clearly. Suppose there are two suspects. No one knows whether they are guilty except themselves. The suspects have two choices: to confess the other suspect or stay silent. The years for them to stay in prison are below. (Figure 5)

A \ B	Confess the other	Silent
Confess the other	15, 15	5, 20
Silent	20, 5	10, 10

Figure 5 The years for the suspects in prison

It is clear that the dominant strategies for A and B is to confess. As a result, The Nash Equilibrium is at the 15-15. (Figure 6)

A / B	Confess the other	Silent
Confess the other	15 / 15	5 / 20
Silent	20 / 5	10 / 10

Figure 6 The Equilibrium of the prisoners (Purple)

However, it is clear that there is another point which both prisoners can have better treatment. At the green point, both prisoners only need to stay in prison for 10 years. It is a situation where competitors will be worse off if they follow the dominant strategies. This is called the Prisoner's Dilemma. The prisoner's dilemma is a paradox in decision analysis in which two individuals acting in their own self-interest pursue a course of action that does not result in the ideal outcome. The typical prisoner's dilemma is set up in such a way that both parties choose to protect themselves at the expense of the other participant. As a result of following a purely logical thought process, both participants find themselves in a worse state than if they had cooperated with each other in the decision-making process. (INVESTOPEDIA)

The prisoners' dilemma has applications to economics and business. Consider two firms; say Coca-Cola and Pepsi, selling similar products. Each must decide on a pricing strategy. They best exploit their joint market power when both charge a high price; each makes a profit of ten million dollars per month. If one sets a competitive low price, it wins a lot of

customers away from the rival. Suppose its profit rises to twelve million dollars, and that of the rival falls to seven million. If both set low prices, the profit of each is nine million dollars. (Figure 7) Here, the low-price strategy is akin to the prisoner's confession, and the high-price akin to keeping silent. Call the former cheating, and the latter cooperation. Then cheating is each firm's dominant strategy, but the result when both "cheat" is worse for each than that of both cooperating.

Coca-Cola / Pepsi	High Price	Low Price
High price	10m / 10m	7m / 12m
Low Price	12m / 7m	9m / 9m

Figure 7 Profits of two firms

Arms races between superpowers or local rival nations offer another important example of the dilemma. Both countries are better off when they cooperate and avoid an arms race. Yet the dominant strategy for each is to arm itself heavily.

Can "prisoners" extricate themselves from the dilemma and sustain cooperation when each has a powerful incentive to cheat? If so, how? The most common path to cooperation arises from repetitions of the game. In the Coke-Pepsi example, one month's cheating gets the cheater an extra two million dollars. But a switch from mutual cooperation to mutual cheating loses one million dollars. If one month's cheating is followed by

two months' retaliation, therefore, the result is a wash for the cheater. Any stronger punishment of a cheater would be a clear deterrent.

The following five points elaborate on the idea:

1. The cheater's reward comes at once, while the loss from punishment lies in the future. If players heavily discount future payoffs, then the loss may be insufficient to deter cheating. Thus, cooperation is harder to sustain among very impatient players (governments, for example).

2. Punishment will not work unless cheating can be detected and punished. Therefore, companies cooperate more when their actions are more easily detected (setting prices, for example) and less when actions are less easily detected (deciding on nonprice attributes of goods, such as repair warranties). Punishment is usually easier to arrange in smaller and closed groups. Thus, industries with few firms and less threat of new entry are more likely to be collusive.

3. Punishment can be made automatic by following strategies like "tit for tat." This idea was popularized by University of Michigan political scientist Robert Axelrod. Here, you cheat if and only if your rival cheated in the previous round. But if rivals' innocent actions can be misinterpreted as cheating, then tit for tat runs the risk of setting off successive rounds of unwarranted retaliation.

4. A fixed, finite number of repetitions is logically inadequate to yield

cooperation. Both or all players know that cheating is the dominant strategy in the last play. Given this, the same goes for the second-last play, then the third-last, and so on. But in practice we see some cooperation in the early rounds of a fixed set of repetitions. The reason may be either that players do not know the number of rounds for sure, or that they can exploit the possibility of "irrational niceness" to their mutual advantage.

5. Cooperation can also arise if the group has a large leader, who personally stands to lose a lot from outright competition and therefore exercises restraint, even though he knows that other small players will cheat. Saudi Arabia's role of "swing producer" in the opec cartel is an instance of this. (Dixit, Nalebuff 2008)

Conclusion

After all, the Prisoner's Dilemma is a special situation of the Nash Equilibrium. The situation will can exist in business market between a few competitors. How to avoid such dilemma is the task of the companies. The most efficient solution is to perform it repeatedly.

References

Myerson, Roger B. (1991). “Game Theory: Analysis of Conflict”, Harvard University Press, p. 1. Chapter-preview links, pp. vii–xi.

Shor Mikhael, "name_of_entry," Dictionary of Game Theory Terms, Game Theory .net, http://www.gametheory.net/dictionary/url_of_entry.html

Osborne, Martin J., and Ariel Rubinstein. A Course in Game Theory. Cambridge, MA: MIT, 1994. Print.

Cournot A. (1838) Researches on the Mathematical Principles of the Theory of Wealth

INVESTOPEDIA, “What is Prisoner’s Dilemma”, http://www.investopedia.com/terms/p/prisoners-dilemma.asp

Avinash Dixit and Barry Nalebuff, "Prisoners’ Dilemma." The Concise Encyclopedia of Economics. 2008. Library of Economics and Liberty. 24 July 2017. <http://www.econlib.org/library/Enc/PrisonersDilemma.html>.