Fibonacci Pattern

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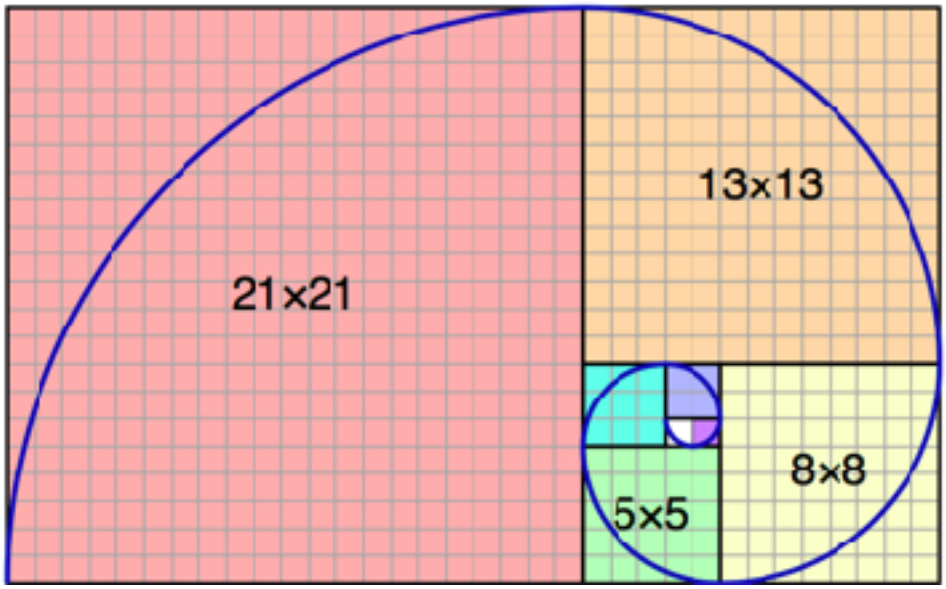
Introduction

Fibonacci series is a sequence of positive integer numbers that follow a certain pattern. In Fibonacci series every number after the second one in Fibonacci sequence is the sum of two number before it. Therefore the sequence goes like this: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55… The sequence is name after a great Italian mathematician Leonardo of Pisa who introduced the sequence in his book Liber Abaci in 1202[1].



FROM WIKIMEDIA COMMONS [2]

Interestingly Fibonacci series is found in many aspects of nature ,such as the number of flower petals and the number of tree branches. More interesting discoveries will be listed in the following paragraphs.

In addition , the ratio between every two continuous number in Fibonacci series is Golden Ratio. For centuries , golden ratio is regarded as harmonic and beauty for no reason. Now, the reason why the golden ratio is symbol of beauty is unveiled by using mathematical methods …

Fibonacci phenomenon in nature

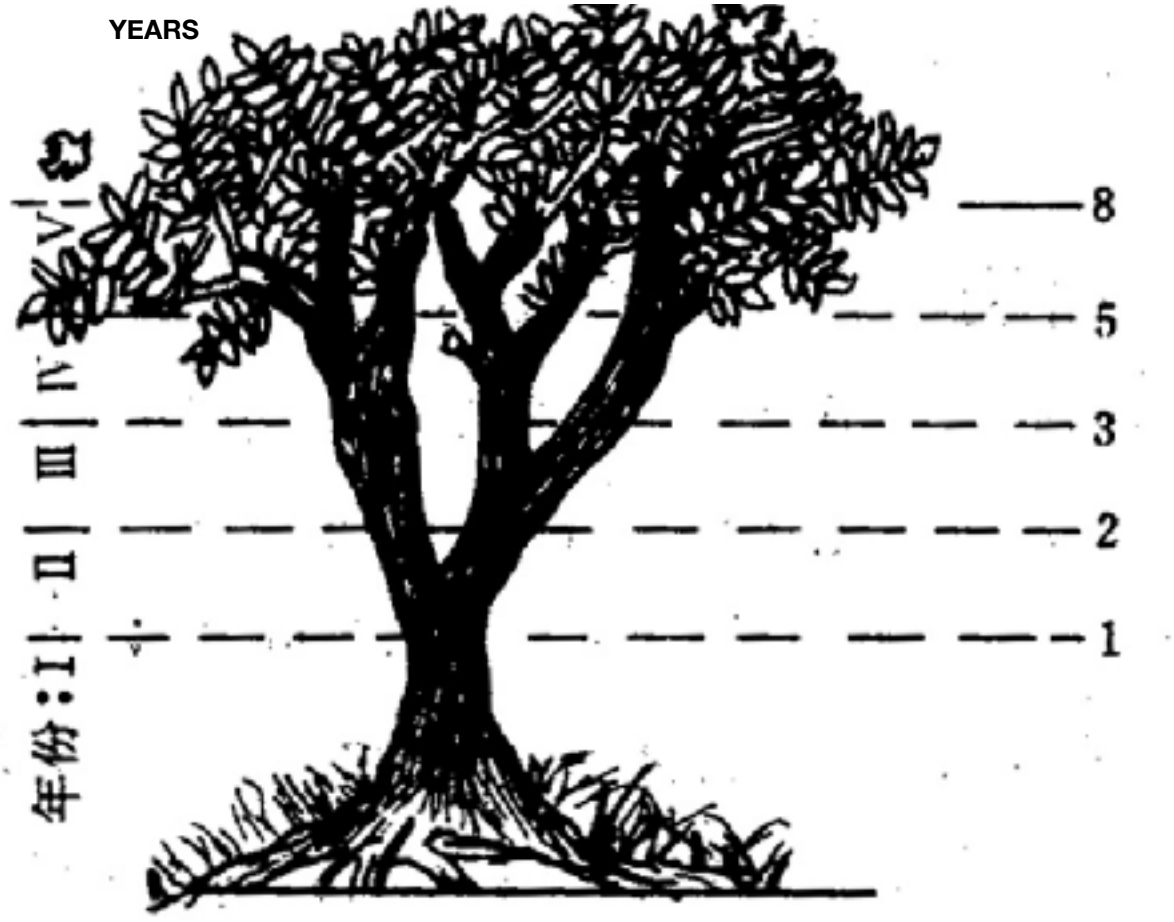
Flowers' petals

According to statistics , Fibonacci numbers appear a lot in botany . Botanists discovered that the numbers of most of the flowers petals on our earth are Fibonacci numbers .For instance , lilies and irises have 3 petals, buttercups and wild roses have 5, delphiniums have 8 and so on[3].

FROM BAIDU PICTURE

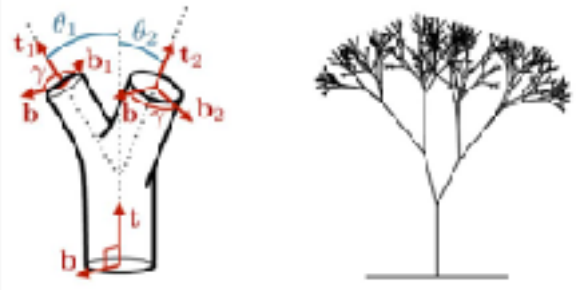
Tree Branches

Additionally ,trees also experience the Fibonacci series in their entire growth. To make it clear , when a tree form a new branch or split into two branches is called a growth point of a tree[4]. At the very beginning of a growth of a tree, it has one branch . As the tree developing ,it reaches a growth point that it form branch, resulting in two growth points. Then, the main trunk develops a new branch, resulting in three growth points. Then the trunk and the first branch make two more growth points, resulting together five branches . The pattern continues, same as the Fibonacci pattern.



Moreover , Leonardo da Vinci once observed tree trunk follow a certain pattern when it splits branches . What Leonardo discovered similar to Fibonacci series which is the sum of the first two objects equal to the third one.

The rule says that the total thickness of the branches at a particular height is equal to the thickness of the trunk. [4.1] When a tree's trunk splits into two branches, the total cross area of those secondary branches will equal the cross area of the trunk. If those two branches each split into two branches, the sum of the cross area of the four new branches equal the area of the cross area of the trunk. Same continues .



[4.1]

Seed spirals

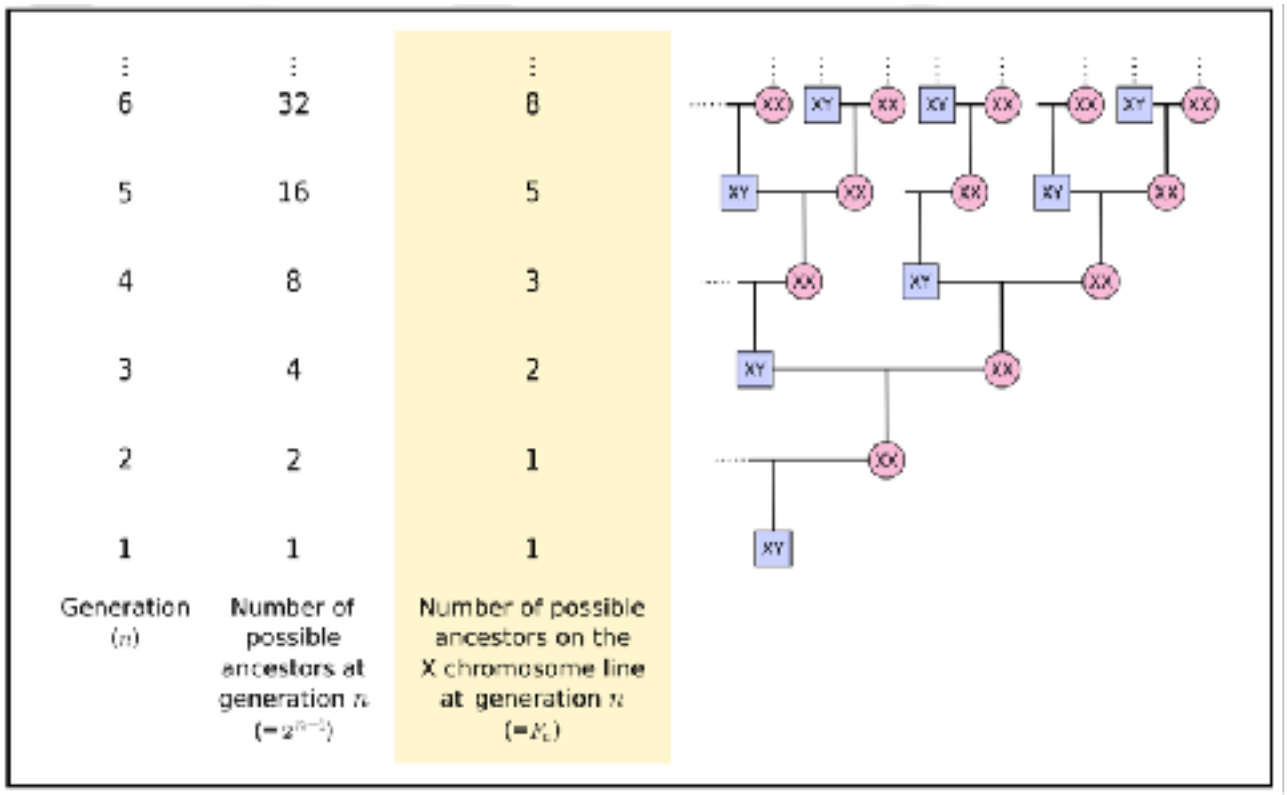
Moreover, some plants have spiral rotating in two opposite directions . For example, in the center of a sunflower, there are spiral rotating left and right. Interestingly, the sum of one spirals in one direction is a Fibonacci number. What is more, the total spirals in the other direction is also a Fibonacci number. And both number of spirals in the opposite direction are two adjacent Fibonacci number. [5] Same manner is found in pinecones, pineapples and cauliflower. Researchers have been growing their own sunflowers and counting the spiral patterns. A study published today in *Royal Society Open Science* reports that after verifying the counts from 657 flowers [5.1], more than four in five of the flowers had Fibonacci spiraling patterns or near-Fibonacci sequences.

[6]

Human X chromosome inheritance

In a given ancestral generation, the number of possible ancestors on the X chromosome inheritance line is same as the Fibonacci series . [7] Suppose a male, Tom, has an X chromosome which got from his mother, and a Y chromosome which got from his father. Tom's X chromosome came from a single parent. Tom's mother got two X chromosome from her mother and father. Therefore we can say that the grandparents gave Tom three X chromosome. The maternal grandfather got his X chromosome from his mother, and the maternal grandmother got X chromosomes from both of her parents, so three great-grandparents contributed to Tom's X chromosome.

Same pattern continues, following the Fibonacci sequence.



FROM WIKI

Fibonacci Series and Golden Ratio

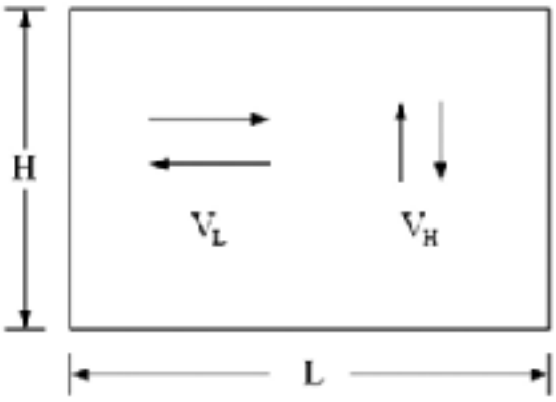
It is exiting that *the ratio between every adjacent number after the second number in Fibonacci series* equals to golden ratio which is approximately 0.618.

For centuries , the great proportion is claimed to be the symbol of harmony and beauty. For example , a rectangle with a ratio of length and width as golden ratio is considered a golden shape. Many artists and architects applied the proportion to their works, such as the Parthenon in Athens and Leonardo da Vinci's painting Mona Lisa.

Golden Ratio Shapes is Easy to Read by Human Eyes

Here comes the question : Why the golden ratio is so appreciated by human eyes? The answer is that human’s eyes can scan the golden ratio shapes with greatest ease.

Consider now the area H × L shown [9]. The shape of the image is free to change. The eye must scan the rectangular area h × l, in order to do it with the least effort , which is in the shortest time.



Human eyes scan the image completely, horizontally and vertically. [10-20]The horizontal sweep covers the length l with the average speed V_l (averaged over several saccades). The horizontal sweep time is $t_l = l/V_l$. The vertical sweep covers the distance h with the averaged speed V_h and time $t_h = h/V_h$.

The total time required to scan the image is of order $t = l/V_l + h/V_h$. The area of the image is fixed ($a = hl$), but the shape of the image is not. The total time is $t = l/V_l + a/(lV_h)$, and it is minimal when $l = (aV_l/V_h)^{1/2}$, which represents this rectangular shape:

$$\frac{L}{H} = \frac{V_L}{V_H}$$

FORMULA 1

The shape of the image is critical to how the image is understood and read. And when the image is shaped according to the formula 1, the horizontal sweep takes just as long as the vertical sweep,

$t_l = t_h$ (2)

In the present case, if $L \geq H$, then$V_l \geq V_h$, such that $t_l = t_h$ means that the time to scan long and fast must be the same as the time to scan short and slow.

Moreover , l/h must greater than 1 to have the ratio V_l/V_h greater than 1. It is proved that human scan things easier on the horizontal than on the vertical. Here is an simple experiment you can do: to scan horizontally is easy, while to scan vertically you have to nod your head which cost a lot of more effort.

The eye mechanics contains information on horizontal eye movement (V_l), but not on vertical movement (V_h) [21][22].

Like the l/h ratio in formula 1, the construction of human's two eyes on a horizontal axis is a designed feature. The horizontal orientation of the eye-eye axis make the transportation of visual information from our horizontal environment to the brain more quickly .

Summary

Fibonacci numbers is found in the natural world. The numbers appear in nature often enough to prove that they reflect some naturally occurring patterns. For plants, this means have the maximum exposure for sunlight or maximum seed arrangement. Fibonacci series also closely related to the golden ratio which reflects the appreciation of the beauty of human and the operation of human organs. Fibonacci pattern is such a intriguing phenomenon and even the pattern has been studied for a longtime, it still has much unknown that waiting to be unveiled.

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