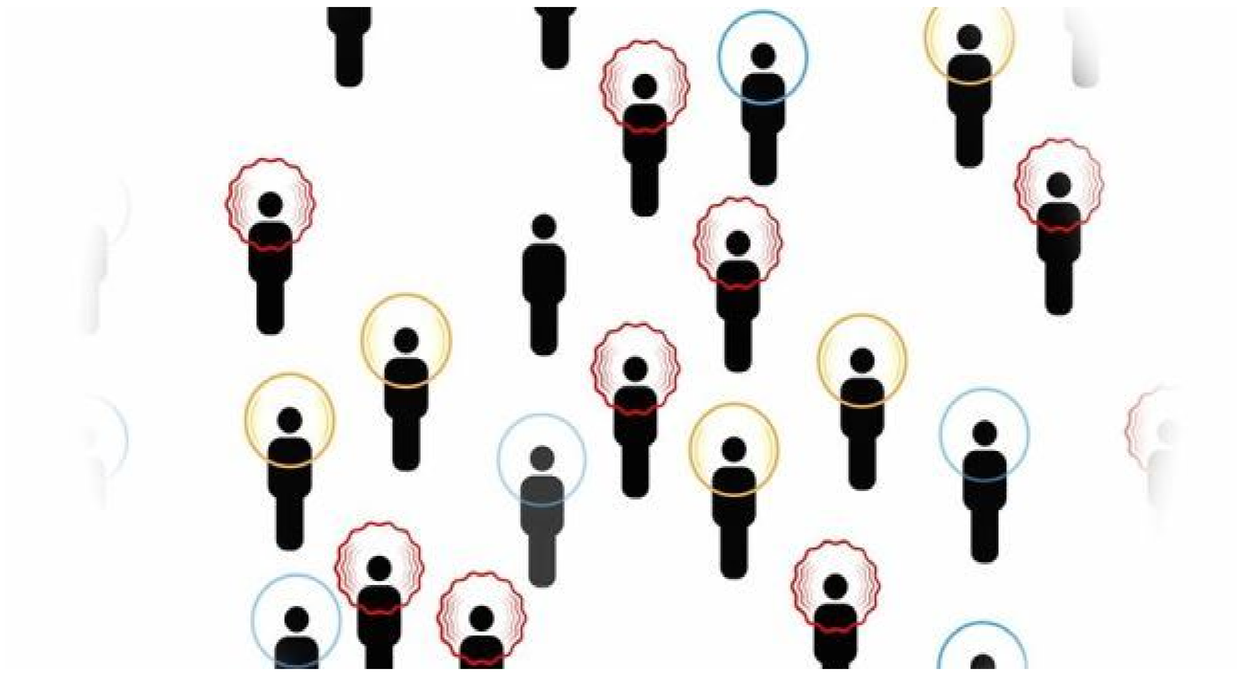
# Disease Transmission through a Community



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Math 190S

Summer 2017

## Introduction

The thought of a disease entering your body at any random moment can send chills down your spine. It can enter from all different places, hair, food, water, blood, and so many more sources. How it travels through your body can be even scarier to think about, starting with one cell, then spreading through the populous like wildfire if not treated within a certain period of time. It may spread and focus on only a few parts of the body, or may look to exterminate all regular cells and replace them with diseased copies. After, or even during its time with the host, it will move on to others and may advance into even more severe types. There is so much behind the science and mathematics of how one spreads, and strongly assists us in how we are able to control and eradicate these entities of harm.

## How diseases expand

There are many ways a disease can spread from one to another, your biggest concern is physical contact with a living animal, including humans. Of course, you can obtain a disease by touching another person or vice versa, being the most common type of transmission. You also have to worry about being scratched or bitten by an animal, and even domesticated house pets can have contagious diseases that are fatal to you. Something as harmless as scooping a cat's litter from a box can spread an infection to you. A form of something between indirect and direct contact is through droplets, which can include bodily fluids from eyes, nose, or the genitalia. The last type of diseases are vector-borne, meaning that living creatures are able to transmit the disease into you, not from touching it.

This leads us to contracting a disease from an indirect source, through air, food, water. It's like direct contact, except you are not contacting a living being. Diseased particles can fly through the air or flow through the water. These airborne and waterborne diseases give themselves an advantageous entryway, right into the mouth or nose. Door knobs, restroom surfaces, as well as computers may hold diseases on the surface, clinging onto your skin and travel to the hairs and make its way inside. If you are infected from food or water, you are a victim of fecal-oral transmission which is caused through poorly cooked food and water that isn't filtered. There is an easy way to highly reduce your risk; simply cook food the way it should be and filter drinking water.

## The SIR Model for spread of Disease

After understanding how a disease grows, it's time to familiarize by how much. The Susceptible-Infected-Recovered (SIR) Model for Disease Spread is a model invented for mathematically calculating the basics of how a disease will spread through a given population of infected, recovered, and susceptible individuals. I classify this as 'basic' because the model is suitable for an isolated town, and not a big city like New York or Phoenix. First, we are going to create a couple of variables representing the entire population, starting with the initially infected total of the population. This will be represented as "$I_t$". The next variable will represent the people who are able to contract the disease, but not yet infected, the susceptible "$S_t$". All that should be left is the healthy populous that is no longer susceptible to contracting the disease and assumed immune for life. The recovered will be represented as "$R_t$". The "t" in parenthesis stands for time measured in days. All of these variables vary with time in the sense that they may change within a day's time. These will later become fractions, as they are all one population.

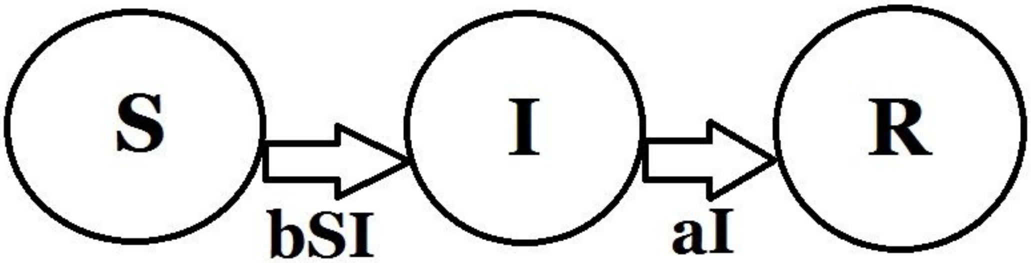
## The Discrete Dynamic Model

A discrete dynamic model means that all initial variables are limited, and no one leaves or enters, and no one dies or gets birthed. Before going into the equation, a couple assumptions are in place. We will be ignoring birth rates, deaths, and immigration for now because we are looking at a short term disease. The total population is represented by "N". Therefore, the population is always the same number since no one is born, no one comes from another country, and no one has or will die:

$$I(t)/N + S(t)/N + R(t)/N = N/N = 1$$

The only way a susceptible can leave the group is by becoming infected. The time-rate of change for susceptibles is dependent on the number of susceptibles, the number of infected, and the amount of contact between the two.

Assume each infected individual makes a certain number of contacts per day, and not all of those contacts are with susceptibles. Another assumption we can make is that the disease is temporary, and is deemed eradicated once the populous reaches full recovery, and never to come back in another wave. Each contact to a susceptible will spread the disease by how many contacts they've made in a day. This will be represented as "b". We can also assume that not every susceptible will get infected overnight because of the infection rate, and because maybe a small number of people are infected. So, the disease must spread with the infection rate, the infectious and the susceptibles, making for the equation: b*S*I. For calculating the number of infectives recovered per day, we must introduce yet another variable that shows the recovery rate, call it “a”. The only other variable for the expression of recovered peoples is the infectious, so the equation will just be: a*I.




For showing a change in the susceptible group, it will go from day one to day 2, so we are just going to add 1 to day 2 to show the change: $S_{t+1}$ - $S_t$. This means that the other side should be equal to our previous equation b*S(t)*I(t), right? Well, it would be true, if the same number of susceptibles joined in on the same day or night, but they aren’t, so the growth rate of the disease weakens because there are a lesser number of susceptibles to infect, making the growth rate negative, making the equation:

$$S_{t+1} - S_t = -b * S_t * I_t$$

About the change in infectious to recovered, we will be applying the same idea we did with S(t). We will be subtracting the day 1 value from the day 0 value: $I_{t+1}$ - $I_t$. When a susceptible gets infected, the number of infected people increases, making the other side of the equation $b*S_t*I_t$. However, a number of infected is already changing because of the recovery rate, so we are going to subtract the side by $a*I_t$. This makes for our second full equation of:

$$I_{t+1} - I_t = b * S_t * I_t - a * I_t$$

Finally, we have our last dynamic equation, the change in recovered. This number will never decrease and will always increase by the recovery rate. So, making it easy, our final equation is:

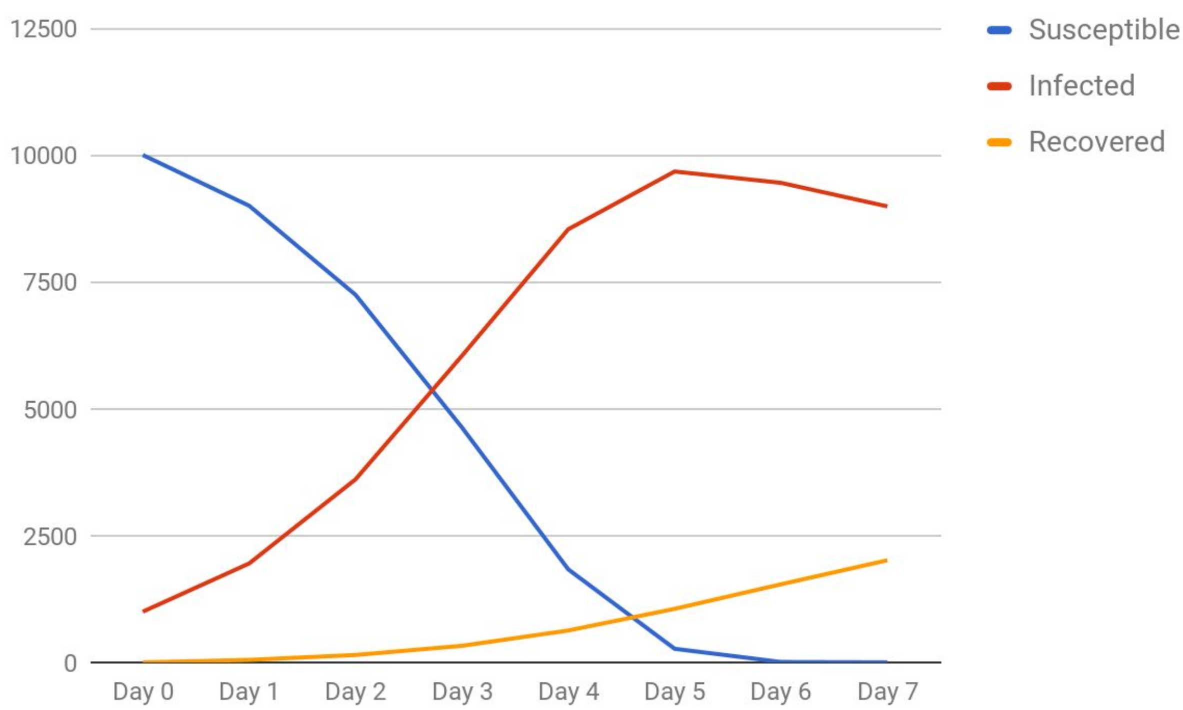
$$R_{t+1} - R_t = a * I_t$$

**<u>Applying to a the discrete system</u>**

The last three equations are the discrete dynamic system for the SIR model. Now, we must test this model, apply it to a population, and give values to our two unknown variables "a" and "b". For an example, let's make a = 0.05 and b = 0.0001, for initial conditions, we will make our susceptible population 10,000 and our infected population 1,000, and no one is cured yet, so our total population is 11,000 people.

Starting from the initial day, t=0, the change of susceptible to infected people is

$b * S_o * I_o$, which will be 0.0001 * 10,000 * 1,000. This totals to 1,000 new infectious individuals. Our recovered population is $a*I_o = 0.05 * 1000$, meaning only 50 people recovered that day, and as one day passed, there are 1000 less susceptibles, leaving 9,000 left to infect. As the recovered population increases, infected people decrease, in this case, by 50, adding 50 to recovered and leaving infected with 1,950. At the end of the day, 9,000 people are left to be infected, 1,950 are already infected, and 50 have recovered.




The chart above shows the spread of the disease using our system of equations over a week's time. For clarification, the total population is 11,000 people, and not 10,000 people. Now, this is somewhat far from a real situation, as no one is born, no one has died, and no one has left or entered the population. However, it is a start to evermore expanding ideas of what to integrate into the mathematics of spreading diseases. Vaccination is a key factor that contributes to the spread of disease and will give more insight to calculations of spread.

## Vaccination

One way to assist the halting of spread of the disease and possibly prevent an epidemic  is to vaccinate people. A vaccine is also beneficial not just for you, but the people around you, as you can no longer be a carrier for the disease. If this were to take effect, it would make sense to take them out of the model entirely instead of put them into the recovered section. This is because it makes it easier to use the recovered pool to identify who contracted the disease and who did not. This means decreasing the model by a certain amount each day but keeping the infectious and recovered at a constant. In this case, it would be possible to vaccinate a couple hundred people from the disease. But, the disease would still get an immense amount of susceptibles either way. Vaccination would have to be received early in order to have an effect on how many susceptibles would be infected, not just start when the disease spreads. The problem then becomes the behavior of the humans, as some may decide not to take the vaccine and continue to spread the disease.

Let's take a short look at the whooping cough vaccine in the late 1970's to early 80's. Years following up to these dates, the vaccine was widely used by many around the world. However, whooping cough then became a rare case because children were vaccinated at a young age, making it near impossible to contract it, and it was almost unheard of. There was then a theory by someone that the vaccine caused a rare side effect causing grave brain damage which was proven false in later years. As for the present, people lost their trust in the vaccine and the proportion of vaccinated people dropped rapidly.

After a couple of years, and disproving the theory, people regained their trust with the vaccine. New and improved vaccines from many countries start to appear into the mix and they are soon administered to the people. In the 2000's, we start to see whooping cough appear again in teenagers and young adults and people are slow to realize it. Usually whooping cough is a disease in young children, but,  modern vaccines give better protection, but for a shorter period of time. The infection then decides to attack the older group, as they stop receiving the vaccine. As the disease becomes more common to the older group, they will start to have kids, and when they cough, it will be near their kids, and the vaccine becomes even less effective, sometimes leading to their death. This exact event happened in the US in 2012, claiming to be the "worst whooping cough year for US since 1955."

Immediately, the United Kingdom responded to this crisis in an amazing way during 2012. They offered the vaccine to pregnant mothers, making the child born immune to whooping cough and should last until they are old enough not to be targeted by it. Antibodies from the mother or breast milk obtained by the baby is filled with the vaccine, therefore, the child grows with the immunity. By 2013, over half of all mothers are immunized, as well as their children.

## Conclusion

There is so much about the spread of disease that makes it of the utmost importance to the human race as a whole. We must understand how it infects, who it infects, as any vaccine or possible cure to prevent an epidemic. A disease can spread through many types of transportation, from your food, to water, to surfaces, and even through thin air, it may survive. The SIR model for disease spread is one that is widely used today to calculate the possible spread of disease within given parameters of the area. Vaccines play one of the greatest roles in stopping the spread of disease, as it does not just affect the patient, but everyone around him/her. Seeing what a vaccine can do to a population within a century, it is crucial that we use these antibodies to our advantage and prepare ourselves for evermore greater threats.

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