Pascal's Triangle

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Introduction

Pascal's triangle is a never-ending equilateral triangle of numbers that follow a rule of adding the two numbers above to get the number below. It is named after a French mathematician Blaise Pascal, who used the triangle in his studies in probability theory, in much of the Western world. In fact, Pascal's triangle was discovered by many mathematician centuries before Pascal in India, Persia, China, and Italy. Therefore, there may be a different naming depending on the country, such as in China, Pascal's triangle is often called Yanghui triangle.

History

Pascal's triangle was known by Jia Xian, a Chinese mathematician, in the early 11th. He used the triangle for high-power operation. Then, in the 13th century, Yang Hui further studied and presented the triangle, so the Chinese call Pascal's triangle as Yanghui triangle. Pascal's triangle was included as an illustration in Chinese mathematician Zhu Shijie's Siyuan yujian (1303; "Precious Mirror of Four Elements"), where it was already called the "Old Method."

In Italy, Pascal's triangle is referred to as Tartaglia's triangle, named for the Italian algebraist Niccolò Fontana Tartaglia (1500–1577), who published six rows of the triangle in 1556.

In Europe, until 1623, Blaise Pascal, a French mathematician, discovered Pascal's triangle at the age of thirteen. His books Traité du triangle arithmétique introduced the triangle, and used it to explain the probability theory. The effect was huge

enough that Pierre Raymond de Montmort and Abraham De Moivre used Pascal to name the Pascal's triangle.

Patterns

- 1. Each number equals to the sum of the two numbers above it.
- Each line of numbers is symmetrical, starting from the number one and gradually become bigger from the side to the middle.
- 3. The nth line have n entries.
- 4. The sum of numbers of the nth line is 2^{n-1} . (The horizontal sum double each time.)
- 5. Each line is the powers of eleven.



For numbers bigger than 11⁵, overlap the digits as the figure below.



6. Color the odd and even number in the triangle, it will end up with a pattern the same as Sierpinski triangle.



7. The first diagonal is just "1"s, the second diagonal has Counting Numbers, the third diagonal has Triangular Numbers, and the forth diagonal (not highlighted) has Tetrahedral Numbers.



8. In the second diagonal, the square of a number is equal to the sum of the

number next to it and the number below both of these two numbers.



9. Make a pattern by going up and then along, then add up the values (as illustrated) ... you will get the Fibonacci Sequence



Combinations

One of the most important field in statistic is the use of permutation and combination. By learning those knowledge, it's very easy to solve some practical questions which can be applied in people's daily lives. The information about Pascal's Triangle plays an inevitable role when using combination. By using the general formula of combinations shown below, it is possible for us to calculate any entry in the Pascal's triangle.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

This formula is commonly called "n choose k", and can also be written C (n,k), ⁿC_k or even _nC_k. It is usually applied to the situation that when we need to choose a certain number of items from a large amount of them randomly, we may wonder how many different results we will get. Using the below example to explain, if a person wants to choose 2 books from a total number of 4 books randomly, he/she will get ${}^{4}C_{2}$ which equals to 6 different outcomes. The result "6" we get is just as same value as the number of the second term on row 4 in Pascal's Triangle.

Example: Row 4, term 2 in Pascal's Triangle is "6"

$$\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4!}{2!2!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} = 6$$

Polynomial

Another field in mathematics which can be related to Pascal's Triangle is the knowledge about polynomial. The most common application occurs when people are trying to figure out questions about binomial expansion. Pascal's triangle shows the coefficients in the binomial expansion, which are represented below:

Power	Binomial Expansion	Pascal's Triangle
2	$(x + 1)^2 = 1x^2 + 2x + 1$	1, 2, 1
3	$(x + 1)^3 = 1x^3 + 3x^2 + 3x + 1$	1, 3, 3, 1
4	$(x + 1)^4 = 1x^4 + 4x^3 + 6x^2 + 4x + 1$	1, 4, 6, 4, 1
	etc	

Therefore, from this form, we can draw the general equation about the coefficients of binominal expansion, which is the formula:

$$(a+b)^n = \sum_{r=0}^n C_n^r a^{n-r} b^r$$

Application

The basic characteristic of Pascal's triangle is that the two hypotenuses are made up by the number 1, and other numbers equal to the sum of two numbers above itself. The pattern within the Pascal's Triangle will not only show us the beauty of math but also represent its practicability in our real world. I'm going to explain this aspect from three different perspectives.

With the rapid development of economy, more and more people are interested in speculating in the stock market. The limit-up question of the stock plays an important role in people's daily life. Therefore, one crucial point is: how many limit-up does a person need to experience so that his initial investment could double. There's no doubt that each limit-up will increase 10% investment. So there's a model that the initial investment is a dollar, after a limit-up, the investment will change to:

a+10%a=(1+0.1)a=1.1a.

After the second limit-up, the investment will change to $1.1a+10\%\times1.1a=1.1|2a$ Therefore, after the nth limit-up, the investment will change to 1.1|na dollar. So how many limit-up do we need to double the investment? We can set up a inequality: 1.1|na>2a, which is the same as: 1.1|n>2. Therefore, what's the smallest integer n? By simple deduction: 1.1|1=1.1, 1.1|2=1.21, 1.1|3=1.331, we can find surprisingly that the first three numbers are corresponding to the value within Pascal's Triangle.

Thus, through a series of calculation of the smallest integer n, we can observe that when n equals to 8, 1.1 8>2 which indicates that after 8 limit-up situations, the investment will double.

Apart from the economic perspective, the principle of Pascal's Triangle can also be applied in marble games: A ball is falling down, after hindered by the first-layer barriers, it will keep falling to the second layer and after hindered by the second-layer, it will fall to the third layer and so on. According to the final area the ball falls, the gamer will get different presents within different value. It's common to see the phenomenon that the presents along two sides have higher value than those in the middle and actually this can be explained by analyzing Pascal's Triangle.



Just imagine the oblique lines in the figure above are marbles' tracks and the marble will fall from that "top 1" place. Therefore from the observation we can find that to fall to the first layer, the marble has 1 possibility to go through the left track and 1 possibility from the right track which is corresponding to the number on the graph. In the second layer, there is 1 possibility for the ball to go thought the side way but two possibilities to go through the track in the middle which is labeled number "2". The possibilities of marble's movement are exactly the same as Pascal's Triangle. Therefore by comparing the numbers on the graph, it's easy to infer that the possibilities for the marble to fall into the middle region are the largest (20 possibilities in sixth layer) and the least for both ends (1 possibility in sixth layers). Pascal's Triangle explains why the presents in the middle are most expensive.

Another example of applying Pascal's Triangle is about shortest route. In

Olympics Games, to save time, the audiences always are trying to find the shortest way from one stadium to another one. Suppose A is the first stadium, and B is the second stadium. How many shortest routes can we find under this situation?



Since there are five horizontal lines and four vertical lines from A to B, we need to determine total 9 lines. But if we can determine those five horizontal lines, those four vertical lines then determine. Therefore by calculating 9 combinations 5, we will get the result 126 which refers to 126 routes. In order to relate to Pascal's Triangle, we can first analyses the route from A to D, E, F, M, N, and K. It's very obvious that there's only one way to go from A to D, A to E, A to F, and A to M. The route from A to N can be separated to A to M to N and A to D to N which shows two ways. Similarly, from A to K can be divided to three situations. Therefore we can mark the

1	1	1	1	1	1
1	2	3	1	5	6
1	3	6	10	15	21
1	4	10	20	35	56
1	5	15	35	70	126
R					

number of routes on the previous graph. B . If we rotate this diagram for 45 degrees, we will find the appearance of Pascal's Triangle's pattern.



Conclusion

From the previous introduction about the Pascal's Triangle, I learn about not only the historical background, characteristics, how to apply it in other mathematical field, but the most important thing is that I know the application of Pascal's Triangle in other fields like economic or gaming which is relevant to people's daily lives.

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