Tetris: Can We Play Tetris Forever and Never Lose?

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Introduction:

The tetris is a tile-matching puzzle video game, originally designed and programmed by Soviet mathematician and game designer Alexey Pazhitnov. [4] There were some mathematicians who spent many hours in studying Tetris.

Nevertheless, little is known about the mathematical properties about the game. In the game, players should match different shapes of tails made from squares until the squares fill up one row so they can be canceled. While tiles are randomly given, no one can promise that they can play the Tetris forever and never lose the game. In the real world, human will make mistakes, so it's impossible to play Tetris and never lose. However, in ideal condition, is it possible to "win" Tetris?

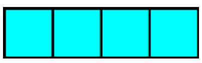
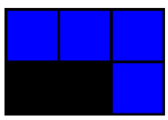
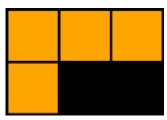
On the other hand

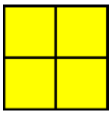
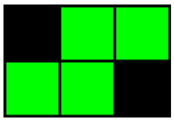
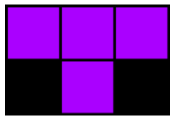
If we are Tetris game machines and we want to defeat a very experienced player, is there a way to defeat the player in a few steps? We know the condition of the game at any time, so we can give the worst shape that can't fit any place of tiles on the floor to drive the player crazy. Can we have a strategy to defeat the player or the player can have an omnipotent way to win the game? Brzustowski proved that there is no winning strategy for Tetris if the computer is aware of and reacting to player's moves. [1]

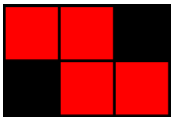
The content of Tetris:

The game field of Tetris is a 10-squares wide and 20-squares high rectangle.

There are 7 kinds of tiles:

I tile:  J tile:  L tile: 

O tile:  S tile:  T tile: 

Z tile: 

https://en.wikipedia.org/wiki/Tetris

Players can see the next tile, can switch the direction and change the position of tiles. Tetris also has gravity effect---when you cancel the squares in the bottom line, squares in the upper line will fall down one square to the bottom line. They will Not just stay there.

History:

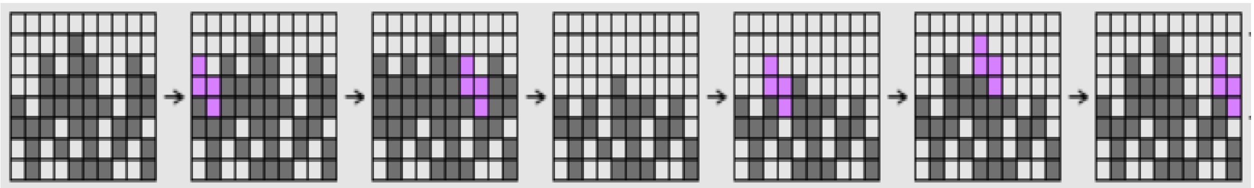
In 1988, John Brzustowski had already tought about this question. He proved that there is no winning strategy for Tetris if the computer is aware of and reacting to player's moves and he gived us a stratage to defeat the player. In his opinion, since the game field is composed of limited squares, so a player has to play in some kind of circulation.[1] In other words, players cancel tiles in a same way, and they just repeat this process again and again.

Thinking in a simple way:

We can analysis the situation from the basic way---if the game machine just gives we one kind of tile, can we use them to form a circulation? If the game machine just gives us I tiles, we can definitely cancel the tiles easily. The same is true with o tiles, J tiles, L tiles, T tiles, Z tiles, and S tiles. So we can conlude that if the game machine just gives us same kind of tiles, we will place the tiles until they are all canceled. We repeat this process. Thus, it forms a loop, and if there is a loop in the game, we never lose the game.

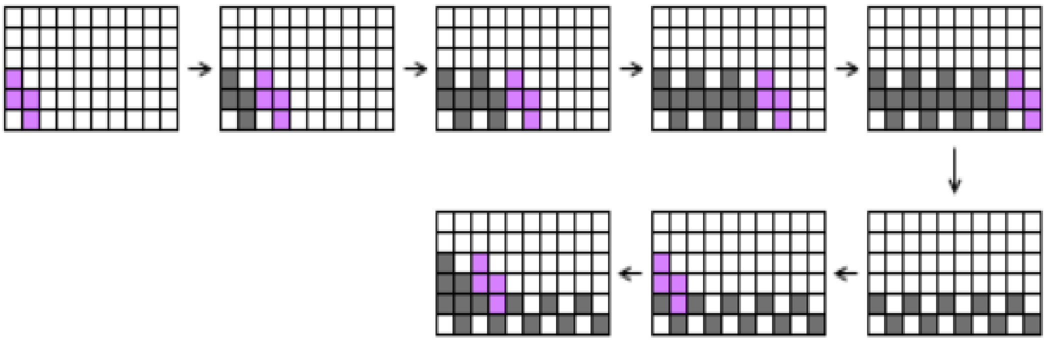
But the situation can be more complex:

The situation mentioned above can just happen when the game field is completely empty. However, in the real game, the game field is impossible to be empty when we play it for several minutes. Then, we can think further: when we play the game, and the situation is really bad, the game machine gives us numerous S tiles. Can we survive in the game? Actually, the answer is "Yes." In the bad situation, the loop can also exist: like the picture bellow, when we place the fifth s tile, the loop occures.



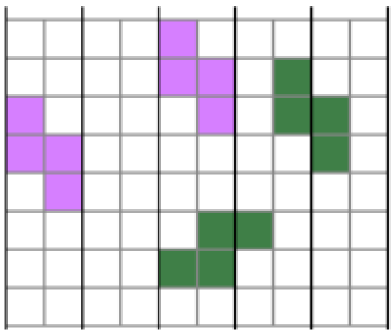
http://www.matrix67.com/blog/archives/2134

John Brzustowski has a concept: he called the rows which are affected by the loop "loop area." [1] For instance, the loop area in the picture above is from line 4 to line 7, because the squares in this lines change. After this, he divided the game area to five lanes. Since the game field is 10 squares wide, a lane is composed of 2 squares. He numbered lanes 1 to 5 from left to right. From pictures above and bellow we can find that every S tile is placed at a lane, and S tiles never cross two lanes.



http://www.matrix67.com/blog/archives/2134

As a matter of fact, as long as we just use S tiles to create a loop, we can have this conclusion. In other words, no matter in which situation, if the game machine gives us numerous S tiles incessantly, and we use these S tiles create a loop, there is only one possible: every S tile only occupys one lanes and completely occupys this lane, like pink tiles instead of green tiles.



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To prove this, we introduce a lemma[5]: first we number the columns of game field from 1 to 10. In a tetris game in which the game machine just provide us S tiles, we will lose the game before we place no more than 120 S tetrominoes if we place S tiles either virtically with their leftmost tiles in an even number column or horizontally in any column.

We can proof the lemma by this means: number the columns of game field from 1 to 10. Let Bx be the total number of cells in the x column, let H_x be the number of horizontal S tiles placed at x column that contribute cells to $(\mathrm{x}-1), \mathrm{x}, (\mathrm{x}+1)$ column, let V_x be the number of verticle S tiles that contribute to x and $(\mathrm{x}+1)$ column in the x column. So we can have

$$B_x = 2V_{x-1} + 2V_x + H_{x-1} + 2H_x + H_{x+1}$$

There is a definition of “death” in the game: when the difference of cell number in two adjacent lines exceeds 20, we are dead in the game. Thus,

$$|B_2 - B_1| = 2V_2 + H_2 + H_3 \leq 20$$

Similarly: $2V_8 + H_8 + H_9 \leq 20$

In general,

$$B_{x+1} - B_x = 2V_{x+1} - 2V_{x-1} + H_{x+2} + H_{x+1} - H_x - H_{x-1} \leq 20$$

$$2V_x + H_x + H_{x+1} \leq 40 \quad (4 \leq x \leq 8)$$

Due to the fact that S tile can’t be placed at 1 and 10 column horizontally, H1=H10=0, and S tile also can’t be placed at 10 column virtically, V10=0.

So we have: $2V_{10} + H_1 + H_{10} = 0$

$$2V_2 + H_2 + H_3 \leq 20$$

$$2V_4 + H_4 + H_5 \leq 40$$

$$2V_6 + H_6 + H_7 \leq 40$$

$$2V_8 + H_8 + H_9 \leq 20$$

$$2\sum_{j=1}^{5} V_{2j} + \sum_{i=1}^{10} H_i \leq 120$$

$$2\sum_{j=1}^{5} V_{2j} > 0,$$

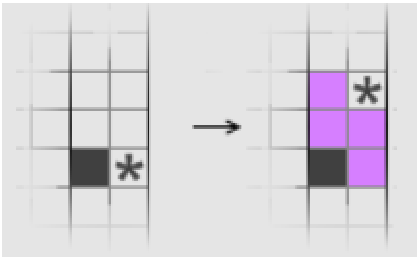
$$\sum_{j=1}^{5} V_{2j} + \sum_{i=1}^{10} H_i < 120$$

This equation tells us if we want to play the game forever by only using S tiles, we have to place each tile virtically and never let them cross lanes' boundary.

But John Brzustowski gived us a deadly way in which players go to the end of game inevitablely[1] :

1. Give players S tiles incessantly untile a loop occure.
2. Give one more S tile.
3. Give players Z tiles incessantly untile a loop occure.
4. Give one more Z tile.
5. Return to 1st step and and repeat this prcess.

Why this is overconstrained? Because after the step one, there will appear a empty square which can't be canceled by Z tile. Although player can have one more S tile, the empty sqaure will appear on the upper line. The only way to cancel the star square is to insert a S tile again, but it will form another empty space which can never be canceled by using Z tile. At this moment, player will receive a lot of Z tiles until entering a loop. This loop is above the previous line which can't be canceled due to the hole in that line. Then, on the left side of this loop area will appear a □■structure, which can't be filled by using S tile. Finally, player receive S tiles again. In this way, lines that can't be filled up will appear in the game field by repeating John Brzustowski's process. In the end, the tiles pile up, and the game is over.



http://www.matrix67.com/blog/archives/2134

While in the real game, the game machine doesn't respond to players' movements. Machines only generate random tiles. The situation mentioned above is very extream, so the pobability of getting a string of S tiles and Z tiles alternatively is small. However, since this probability still exist, we can conclude every tetris game has an ending.

Some defects and unanswered questions:

As I mentioned above, if players don't place the S tile vertically and occupy the whole lane, the game will be over when players place no more than 120 S tiles. The 120 is just a rough estimation. The actually number, I believe, is far less than 120. Mathmaticians haven't found any other solution which can narrow the range of the number of S tiles. The method used in estimating the S tiles doesn't include other shapes of tiles. As the consequence, we still don't know how much tiles we can place before we lose the game. Besides this, we haven't figure out other sequences of tiles which can let a player lose the game definitely and the probability of these sequences.

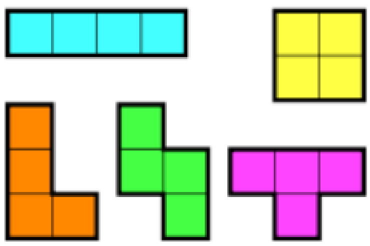
Modern version of Tetris and improvement:

In order to avoid the 100% death situation and improve the quality of the game, the modern tetris revise the program to promise that players will never receive more that 4 Z tiles or S tiles in a row. This is one of the indispensible rule created by Tetris Guildline. All of the official tetris games must follow this rule. Player can also swich the orientation of the tiles when they have touched the bottom line, which increase the chance of survival.[4]

Other interesting problems:

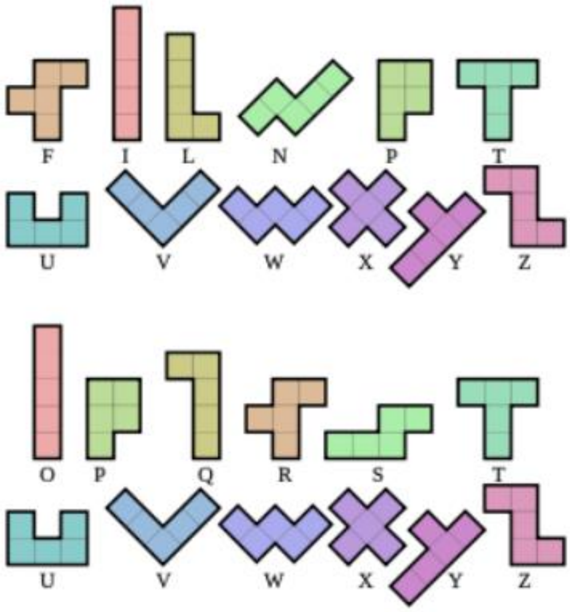
1.Why all of tiles in Tetris are composed of 4 squares?[3]

There is a concept: people called any shape of tiles composed of squares "Polyomino." The tiles composed of 4 squares are called "Tetromino."



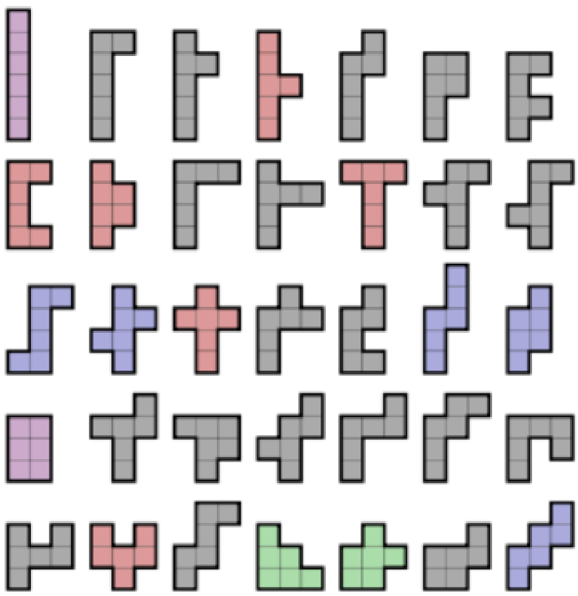
en.wikipedia.org/wiki/Tetromino

Tiles composed of 5 squares are called "Pentomino."



en.wikipedia.org/wiki/Pentomino

Tiles composed of 6 squares are called “Hexomino.



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en.wikipedia.org/wiki/Hexomino

From the pictures above we can observe that when the number of squares increases, the kinds of shapes increase dramatically. One thing which regretful is that the creator Alexey Pazhitnov didn’t give us an answer. I think the most reasonable answer is that in the balance of the complexity and playability of the game, the creator chose to use Tetromino since there are too many shapes in Pentomino. That's also why the game is called Tetris.

2. because every kind of tiles are composed of 4 squares, can we join them together to create a 4×7 rectangle?[2]

No, we can't. We can put the rectangle on the chessboard. Then we find that the rectangle occupys equal number of black and white squares. All kinds of the tiles except T tiles occupy 2 black and 2 white squares on the chessboard. Since the number of white and blace squares occupyed by T tiles are not equal, we can't make up a 4×7 rectangle.

Reference:

1. "Matrix67: The Aha Moments." *Matrix67 The Aha Moments*. N.p., n.d. Web. 17 July 2017. <http://www.matrix67.com/blog/archives/2134>.
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3. 话题的优秀回答者, 曹文雯用户标识儿童教育, and 长天之云用户标识前端开发话题的优秀回答者. "为什么俄罗斯方块中的方块都是由 4 个正方形组成的？." *知乎*. N.p., 18 Oct. 2012. Web. 17 July 2017. <https://www.zhihu.com/question/20540487>.
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5. http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.55.8562&rep=rep1&type=pdf