Some ways to measure distance in astronomy and their history

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Math of Universe

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Introduction:

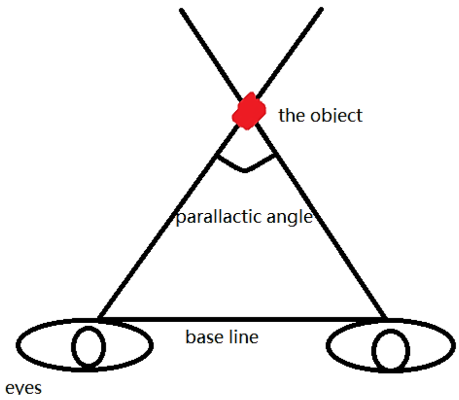
When raising our head to look up to the sky during the night, we can see myriad stars. The distance between stars and us seems so far that even light has to spend many years to travel from stars to reach the earth. The distance is almost unmeasurable to people. However, some astronomers came up with great ideas to measure those extremely long distance. This paper will include some ways used by astronomers to measure the distances between stars and earth.

History

Parallax

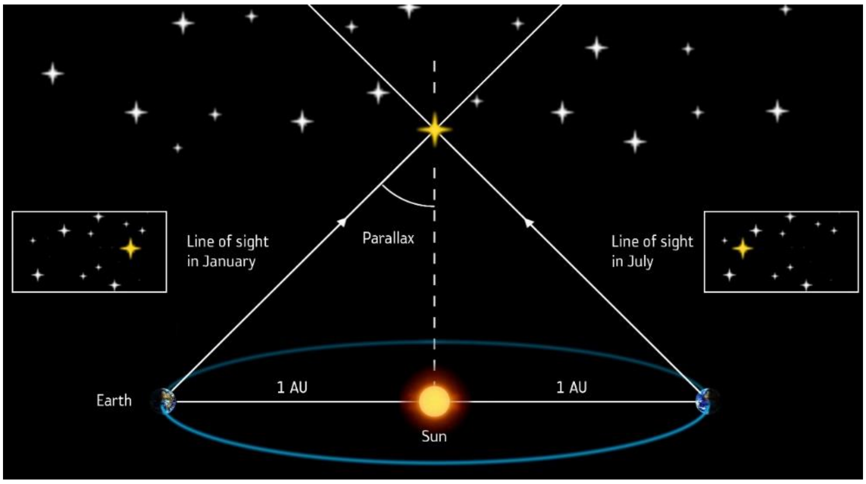
Great ideas usually originate from daily life. In 1838, people found a way to measure the distance---Parallax---from their daily life experience: when people see an object with their eyes, the feeling of distance comes from Parallax. [1] Parallax comes from the Greek word "parallassein", which means "to change". [7] Human eyes separate about 6 centimeters, so the views received by our two eyes are slightly different. Human brain has ability to use Parallax to roughly compute the distance.

However, when the distance is very long, the parallactic angle is small, and we can't measure the distance by using our brain any more.



Astronomies adapt the same way, parallax, to measure the distances of stars which are not too far away from earth. When the earth revolves around the sun, annual parallax appears. First, we define some terms: the base line is the diameter of earth's orbit, the parallactic angle is the angle that forms when we connect the earth, the star we want to observed, and the sun. The parallactic angle is lager when the fixed star is closer to us. Parsec (PC) is a unit which measures distance, and it is acrtually constituted of two words: Parallax and Arc-second. Parsec is the reciprocal of annual parallax: $\frac{1}{p}$.[2] At the time that the annual parallax is 1 angle of second, the fixed star is 1 Parsec from us. $1\text{ Parsec} = 206265\text{ AU} = 3.26\text{ light year} = 3.09\times10^{13}\ km$. [1]

Since 1 angle of second is small, the difference between two sides of this angle can be omited. We can regard this triangle as either right triangle or isosceles triangle.



http://www.esa.int/Our_Activities/Space_Science/Gaia/Parallax

As mentioned above, because P is too small, $\mathrm{R} \approx \mathrm{R}'$, we assume that P=1 angle of second, and the distance from the earth and sun, 1 AU, can be seen as part of perimeter of the circle which radius is R. Then, we can have:

$$\frac{1\,AU}{2\pi R} = \frac{1\,angle\ of\ second}{360\times60\times60\,angle\ of\ second}$$

$$\mathrm{R} = \frac{360\times60\times60\,AU}{2\pi} = 206265\ \mathrm{AU} = 3.26\ \text{light year}$$ [1]

Nevertheless, Parallax has limitations. Since a large portion of fixed stars is too far from the earth, parallactic angles are too small. The paralactic angle of closest fixed star from us, Centaur Centauri, is only 0.75 angle of second[1], and because we observe the stars on the ground, atmosphere disturbs our observation, so the error is large. As a result, when stars' distances exceed 110 PC, scientists use a new method: Spectroscopic Parallax.

Spectroscopic Parallax

In 1914, American scientist Walter sydn Adams and Germany astronomer A.

Kohlschutter worked together to found a new way to measure distance between fixed stars and the earth. [3]They first found that giant stars' spectrums are different from spectrums of main-sequence stars. Observing spectrum of one star, they distinguished the catagory of the star, and then they knew the position of that star in Hertzsprung–Russell diagram and the roughly absolute magnitude of the star according to intensity of spectral lines and Hertzsprung–Russell diagram. Knowing the absolute magnitude and apparent magnitude, they were able to calculate the distance. [8]

1. **Absolute magnitude (M):**

   We assume that we put a star 10 Parsec (32.6 light years) from earth and then measure the luminosity of the star. It reflects the ability of stars to emmit light.[9]

2. **Apprant magnitude (m):**

   It is the luminosity of a star that observed by us on the earth. **m** is affected by stars' luminosity and distance. The scale of Apprant magnitude can be both positive and negative. The brighter the star is, the larger the **m** is. [10]

Principle

We can compare stars to street lumps. As we go closer to one of lumps, we can see the luminosity of the lump becomes brighter. This is beacause the density of light energy is propotional to the riciprocal of the square of distance.

$$\mathrm{E} = \mathrm{cL}/r^2$$

E is illuminence. L is light intensity. r is distance.

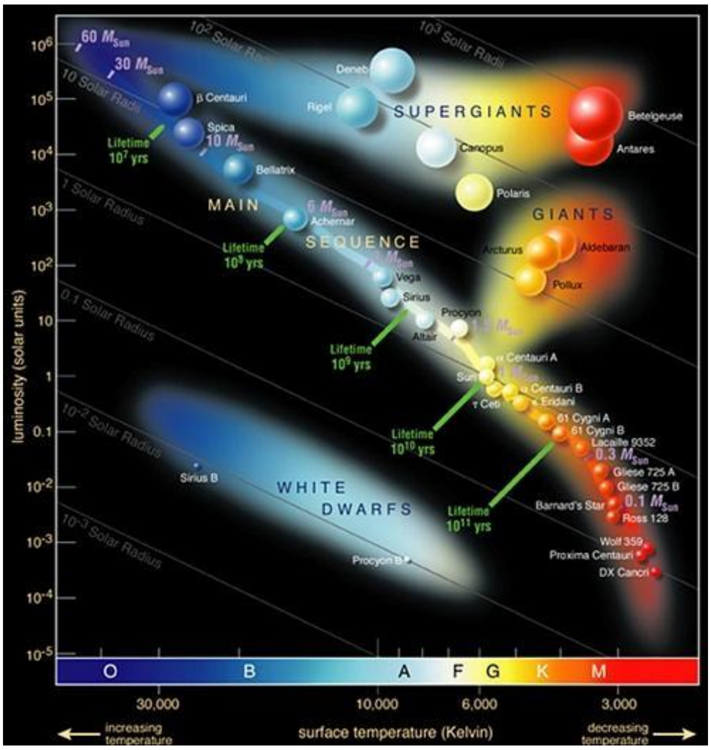
The same is true with fixed stars, due to the huge difference in distance between stars, the dark one among stars may be the most luminous. If we want to know the Absolute magnitude, we move the star 32.6 light year from earth and measure the illuninence again. The ratio between Apprent magnitude and Absolute magnitude can be calculated by computing the ratio between 32.6 light year and the square of true distance. Astronomers find the relationship between Absolute magnitude, Apprent magnitude, 32.6 light year, and true distance. [8]

$$\mathrm{m} - \mathrm{M} = -5 + 5\log D$$

limitations:

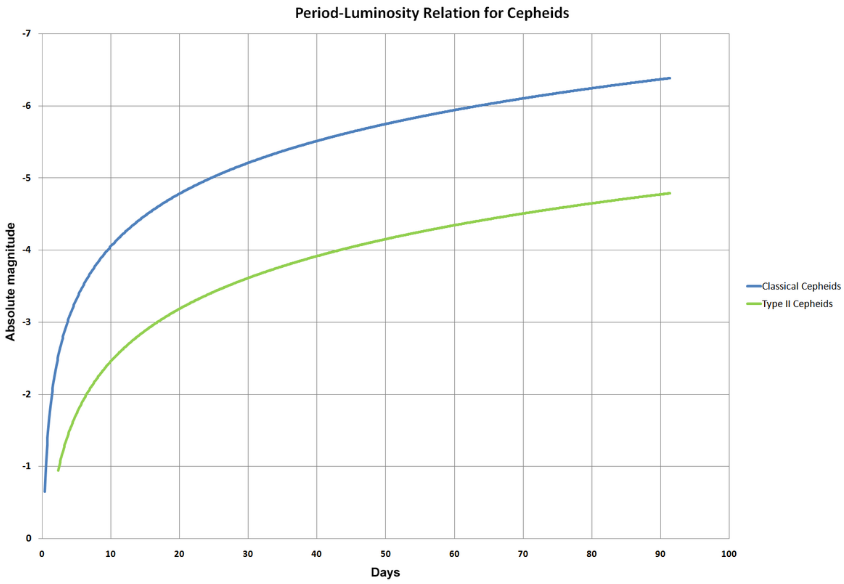
For fixed stars which are very dark, astronomers can't obtain their spectrum, so Spectroscopic Parallax can't be used to measure the distance of these fixed stars.

Hertzsprung–Russell diagram: In 1911 and 1913, Denmark astronomer Hertzsprung and english mathematician Bertrand Arthur William Russell, 3rd Earl Russell come up with the graph respectively. Hertzsprung–Russell diagram is the main tool to study the evolution of fixed stars. Hertzsprung–Russell diagram tells us the relationship between spectral type and lunimosity. The Y axis is luminosity and absolute magnitude; the X axis is spectral type and temperature of surface of stars, which decreases from left to right of the graph. There are 7 kinds of spectral type: O, B, A, F, G, K, M. [11]



Cepheid Variable

In 1908, American astronomer Henrietta Swan Leavitt found 25 variable stars when she studied Large Magellanic Cloud and Small magellanic cloud. Variable stars are fixed stars whose brightness fluctuates. The variation may be caused by a change in emitting light or by something partly blocking the light. So, there are two kinds of variable stars: **Intrinsic variables**, whose luminosity actually changes. **Extrinsic variables**, whose fluctuations in brightness are due to changes in the amount of their light that can reach earth. Variable stars change their luminosity as time passed. About 20% of variable stars are eclipsing binary, which composed of two stars. When one star passes in front of the other star periodically, the light emitted by the other star will be blocked. Most of variable stars change their luminosity because of changes in their volume. Henrietta Swan Leavitt found that there is a relationship between luminosity of variable stars and period of light variation when she investigated thousands of variable stars in Magellanic Clouds. The larger the average luminosity of variable stars is, the longer the period of light variation is. This relationship is called period-luminosity relation, and astronomers calibrated this relationship.



https://en.wikipedia.org/wiki/Classical_Cepheid_variable#/media/File:Period-Luminosity_Relation_for_Cepheids.png

Type II Cepheids are variable stars which pulsate with periods typically between 1 and 50 days, and they are old, low-mass object. In contrast, **Classical Cepheids** are younger, brighter, 4-20 times more massive than the sun, and exhibit regular radial pulsaations with periods of few days to a few weeks.[12]

Having the graph, astronomers found the way to calculate the distance of different Cepheid variable stars to the earth: Astronomers can compute the absolute magnitude by observing period of light variation and using period-luminosity relation graph. Then they record Apprant magnitude of the variable star to compute the distance by using Spectroscopic Parallax.

Limitations:

The range of application is about 5 Mpc (1.6×10^7 $light\ year$). For the galaxies that exceed this distance, we are not able to observe Cepheid variable stars, Thus, Cepheid Variable lose efficacy.

Using Hubble's law to measure distance

In 1924, American astronomer Edwin Powell Hubble confirmed the existence of anagalactic nebula. In 1927, Hubble found that spectra shift exists commonly in anagalactic nebula, and the distance of nebula is proportional to redshift. [4]

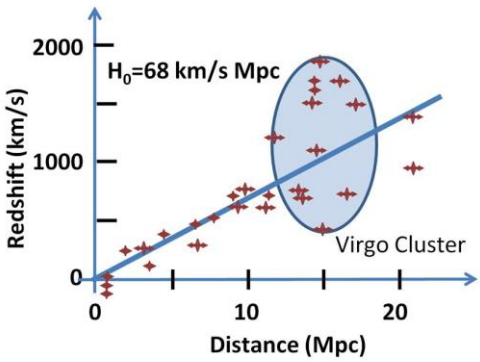
$$\mathrm{D} = \frac{\mathrm{Cz}}{H_0} \text{ or } \mathrm{v} = H_0 D$$

C is light's speed; z **is redshift**, $\mathrm{z} = \Delta\lambda/\lambda$; $H_0$is Hubble constant, which is about 68 km/s Mpc; v is recession velocity. The equation tells us that the longer the distance is, the larger recession speed is. The redshift z is often described as a redshift velocity, which is the recessional velocity that would produce same redshift if it were caused by linear Doppler effect. The redshift velocity can easily exceed the speed of light:

$$v_{rs} = Cz \quad [4]$$

v_{rs} is redshift velocity. The conclusion is that the universe is expansion evenly, and it supports the Big Bang Theory. [5]

Hubble's law can be easily depicted in a "Hubble Diagram" in which the velocity (assumed approximately proportional to the redshift) of an object is plotted with respect to its distance from the observer. A straight line of positive slope on this diagram is the visual depiction of Hubble's law. However, the value of the Hubble parameter changes over time, either increasing or decreasing. [5]



https://en.wikipedia.org/wiki/Hubble%27s_law#/media/File:Hubble_constant.JPG

Doppler effect: the frequency of the wave that received by receiver depends on the movement of receiver and wave source. [4]

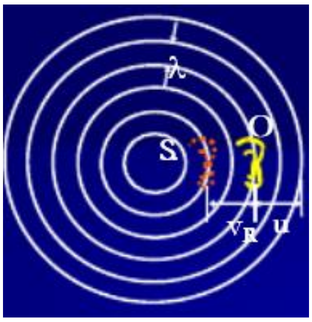
1. We assume the wave source is static with respect to medium, and receiver is moving toward wave source with speed v_r. In unite time, the wave emitted by the wave source travels a distance of u, and receiver travels toward wave source with a distance of v_r. This can be seen as the wave passes the receiver by a distance of $\mathrm{u} + v_r$. In unit time, the number of waves that pass the receiver is

$$\gamma' = \frac{u+v_r}{\lambda} = \frac{u+v_r}{uT} = \frac{u+v_r}{u}\gamma$$

2. It shows us that the frequency of the wave received by receiver changes. In general, we have:

$$\gamma_R = \frac{u \pm v_r}{u}\gamma_S$$

γ_R is the number of waves emitted by wave source in unit time; γ_S is the number of wave received by receiver in unit time. When receiver move toward wave source, the sign is positive; on the other hand, the sign is negative.



3. We assume that receiver(O) is static with regard to medium, the wave source(S) moves with velocity v_s toward receiver. The wave length received by receiver is

$$\lambda' = \lambda - v_S T.$$

The number of waves which pass the receiver is

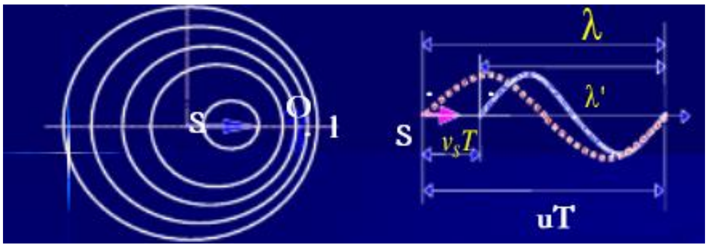
$$\gamma' = \frac{u}{\lambda - v_s T} = \frac{u}{(u - v_s)T} = \frac{u}{u - v_s}\gamma$$

In general, we have:

$$\gamma_R = \frac{u}{u \pm v_s}\gamma_S.$$

4. When both receiver and wave source move relative to medium, the equation is

$$\gamma' = \frac{u \pm v_R}{u \pm v_S}\gamma$$



Doppler effect in light waves: Electromagnetic wave travels with light speed in vacuum, and its speed is constant. Unlike sound waves, the spread of electromagnetic wave doesn't depend on medium. As the result, the equation changes. When the receiver and wave source move in the same straight line in opposite direction, we have

$$\gamma_R = \gamma_S \sqrt{\frac{1 \pm \frac{v}{c}}{1 \mp \frac{v}{c}}}$$

When receiver and wave source get close to each other, the relative velocity(v) is positive. On the other hand, relative velocity is negative. For the former mentioned of two, the frequency of electromagnetic wave received by receiver is higher than the frequency of the wave emitted by wave source, which is called blue shift. For the later one, it is called redshift. When wave source moves away from the receiver in very low speed relative the speed of light, the equation can be written by [5]

$$\frac{\gamma_S}{\gamma_R} = \frac{\lambda'}{\lambda} = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} \approx \frac{1 + \frac{v}{2c}}{1 - \frac{v}{2c}} \approx 1 + \frac{v}{c}$$

And Redshift z equals to:

$$\mathrm{z} = \frac{\lambda' - \lambda}{\lambda} = \frac{v}{c} \quad [6]$$

Knowing redshift z, we can compute the distance.

Limitations: It's very hard to measure Hubble constant precisely. In order to obtain Hubble constant H_0, astronomers have to measure the distance of galaxies that are 100 Mpc or further from the earth. However, this seems impossible for our existing technology to obtain the position of those celestial bodies.

Conclusion:

From a little portion of the history of measuring the distance, we can see the trend: The distance which can be measured is become longer and longer, and some methods is based on former method. As humans' understanding of the universe becomes deeper and deeper, astronomers will able to predict the ultimate fate of universe.

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[12] "Media." Wikipedia. Wikimedia Foundation, 30 June 2017. Web. 21 July 2017.

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