Strategy and Effectiveness: An Analysis of Preferential Ballot Voting Methods

Max Tabachnik
Advisor: Dr. Hubert Bray
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Contents

- Properties
- Voting Systems
- **Systems and their Properties**
- **The Crowding Problem**
- **Agreement and Wins Analysis**
- Conclusion
- Suggestions for Further Research

Properties

- Universality
- Monotonicity
- Independence of Irrelevant Alternatives (IIA)
- Citizen Sovereignty
- Non-dictatorship
- The Majority Criterion
- The Condorcet Condition
  - Condorcet winner
  - Condorcet loser

Voting Systems

- Single Vote Plurality (SVP)
- Instant Runoff Voting (IRV)
- Borda Count (BC)
- Instant Runoff Borda Count (IRBC)
- Least Worst Defeat (LWD)
- Ranked Pairs (RP)

Systems and their Properties

Voting Systems	Universality	Monotonicity	IIA	Citizen Sovereignty
SVP	Yes	Yes	No	Yes
IRV	Yes	No	No	Yes
BC	Yes	Yes	No	Yes
IRBC	Yes	No	No	Yes
LWD	Yes	Yes	No	Yes
RP	Yes	Yes	No	Yes

Voting Systems	Non-dictatorship	Majority criterion	Condorcet winner criterion	Condorcet loser criterion
SVP	Yes	Yes	No	Yes
IRV	Yes	Yes	No	Yes
BC	Yes	No	No	No
IRBC	Yes	Yes	Yes	Yes
LWD	Yes	Yes	Yes	No
RP	Yes	Yes	Yes	Yes

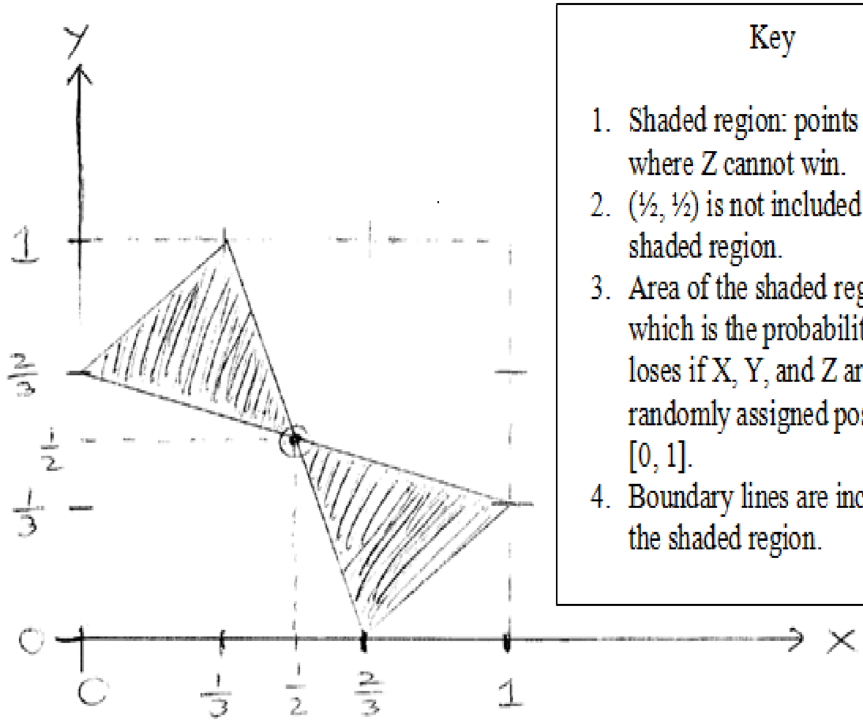
The Crowding Problem

- The ability for n-1 candidates to prevent the nth candidate from winning
- Incentivizes voters against revealing true preferences by not “throwing away” votes
- E.g. U.S. two-party system
  - Ralph Nader
- What methods are most susceptible to crowding?

Crowding Assumptions

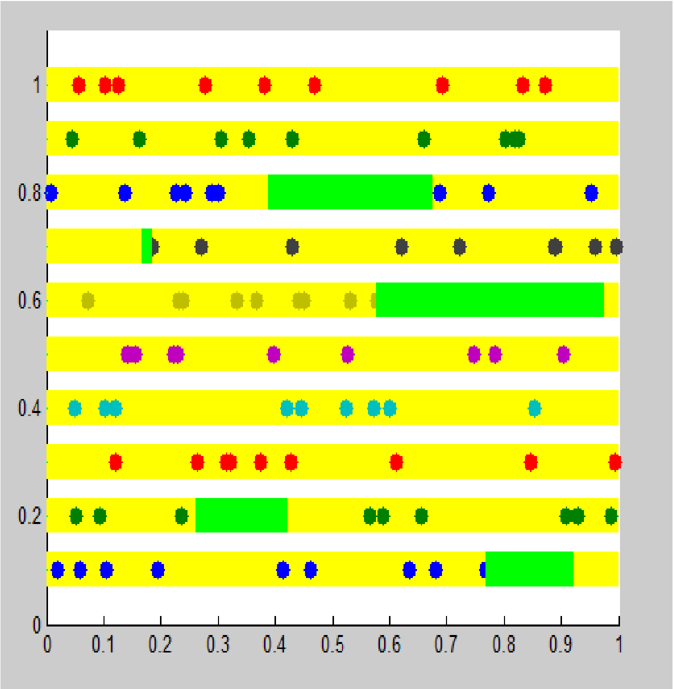
- Voters are uniformly distributed along unit interval [0, 1] and each voter has one vote
- $X_1 \ldots X_n$ are candidates that choose unique positions on [0, 1]
- $X_1 < X_2 < \ldots < X_n$, but in general order is arbitrary
- $P(X_i)$ represents the percentage of the vote candidate Xi receives
  - $P(X_i) = X_i + 0.5*(X_{i+1} - X_i)$ for $i=1$
  - $P(X_i) = 1 - X_i + 0.5*(X_i - X_{i-1})$ for $i=n$
  - $P(X_i) = 0.5*((X_i - X_{i-1}) + (X_{i+1} - X_i))$ for $i \in [2, n-1]$
- There can only be one winning candidate (no ties)

Crowding: Single Vote Plurality

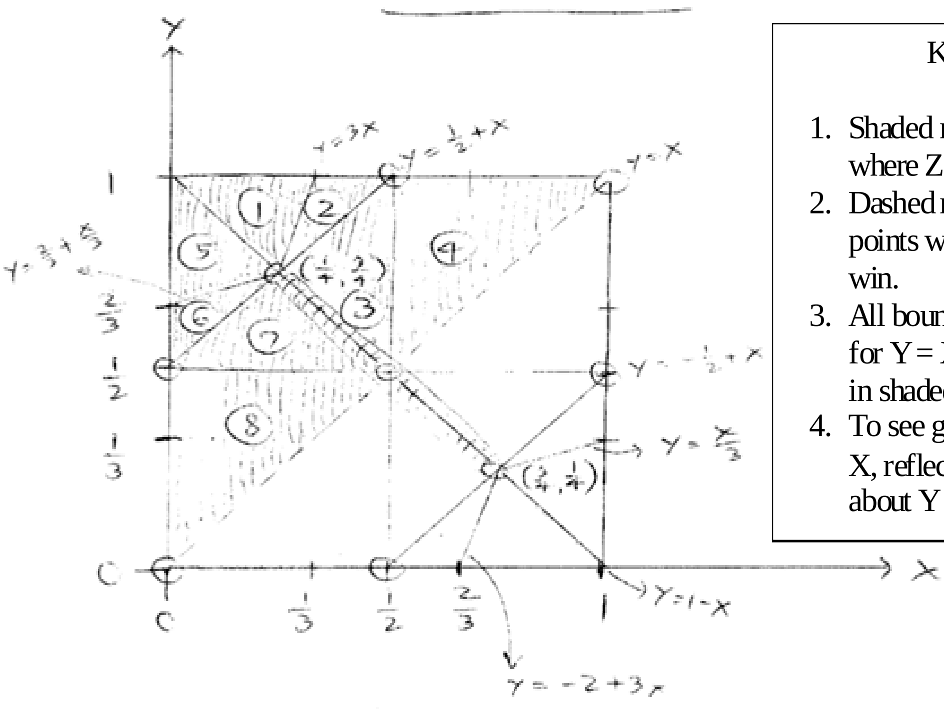


Key

1. Shaded region: points (X, Y) where Z cannot win.
2. (½, ½) is not included in the shaded region.
3. Area of the shaded region is 2/9, which is the probability that Z loses if X, Y, and Z are randomly assigned positions on [0, 1].
4. Boundary lines are included in the shaded region.

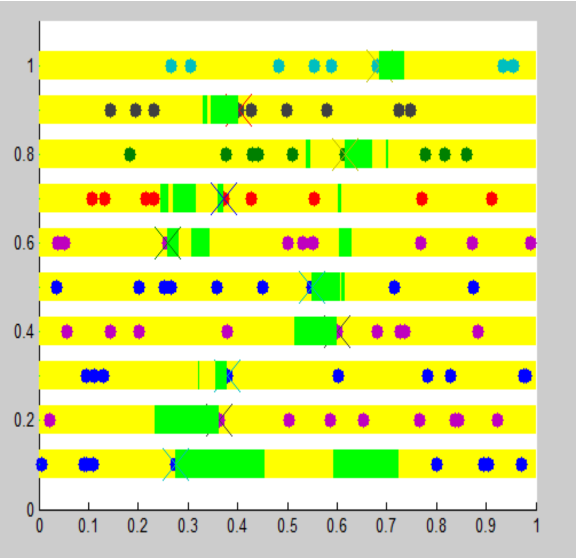


Crowding: Instant Runoff Voting



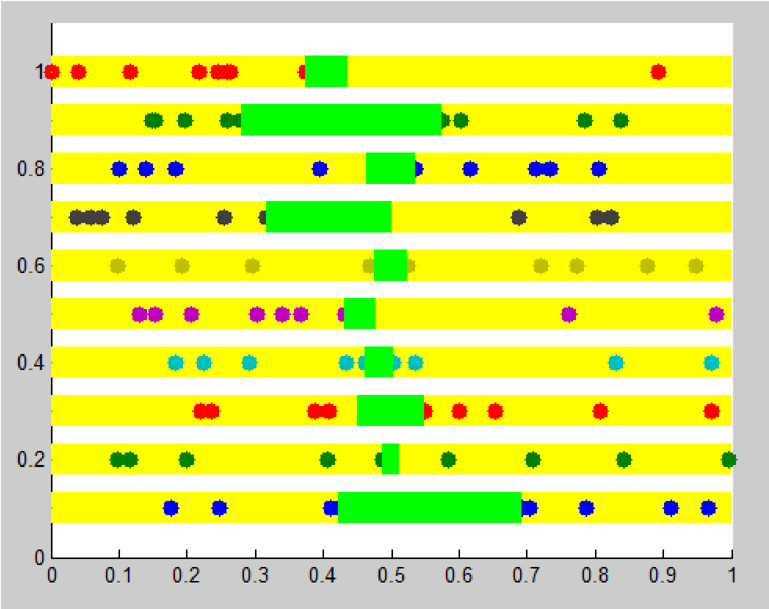
Key

1. Shaded regions: points where Z can win.
2. Dashed middle line: points where Z cannot win.
3. All boundaries except for Y = X are included in shaded regions.
4. To see graph for Y < X, reflect all regions about Y = X.

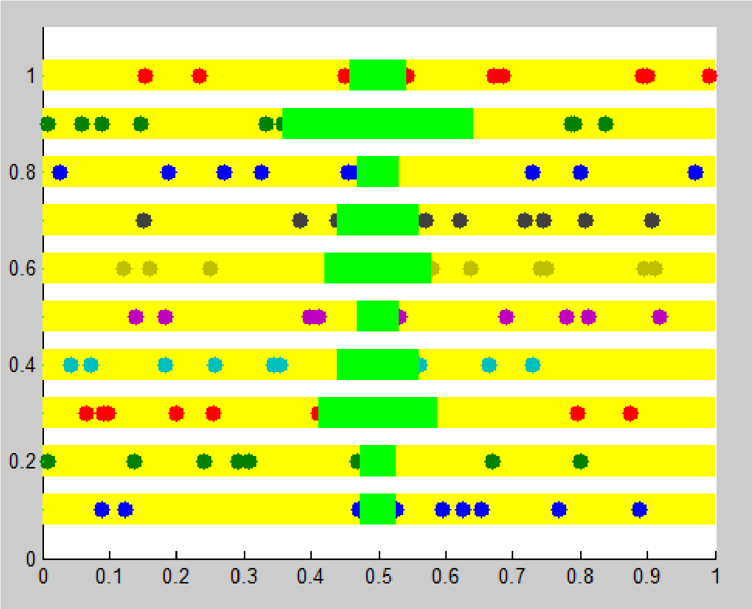


Crowding: Borda and Instant Runoff Borda

Borda Count



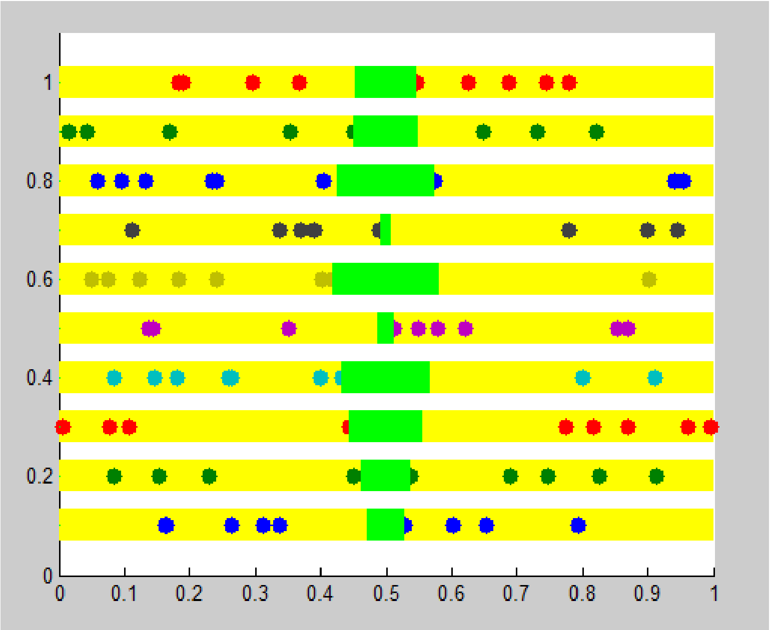
Instant Runoff Borda



Crowding: LWD and Ranked Pairs

Least Worst Defeat

Ranked Pairs



Crowding Summary

Crowding Summary: 1,000 sample elections

Number of Candidates	SVP %	IRV %	Borda %	IRBC %	LWD %	RP %
2	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
3	21.40%	0.10%	0.00%	0.00%	0.00%	0.00%
4	26.90%	0.00%	0.00%	0.00%	0.00%	0.00%
5	34.70%	0.00%	0.00%	0.00%	0.00%	0.00%
6	35.20%	0.00%	0.00%	0.00%	0.00%	0.00%
7	42.20%	0.10%	0.00%	0.00%	0.00%	0.00%
8	43.10%	0.00%	0.00%	0.00%	0.00%	0.00%
9	46.00%	0.00%	0.00%	0.00%	0.00%	0.00%
10	47.10%	0.10%	0.10%	0.00%	0.00%	0.00%

Crowding Summary

- Susceptible to crowding:
  - Single Vote Plurality
- Generically not susceptible to crowding:
  - Instant Runoff Voting
    - Winning strategies are disjoint
- Virtually no possibility of crowding:
  - Borda Count
  - Instant Runoff Borda
  - Least Worst Defeat
  - Ranked Pairs
    - All above methods favor non-disjoint, centrist strategies

Agreement and Wins Analysis

- Compare LWD and Ranked Pairs with Borda (control) using random elections in MATLAB
- How often do these methods agree on a winner?
- When they disagree, how often do the winners from each method win head-to-head against other method winners?
- How often does a Condorcet winner exist?

Three Candidate Case

- LWD and Ranked Pairs winners always either beat or tie Borda winner
  - Strength of Condorcet winners and methods
- LWD and Ranked Pairs always agree
  - Regardless of whether Condorcet winner exists!

Borda vs. LWD and Ranked Pairs

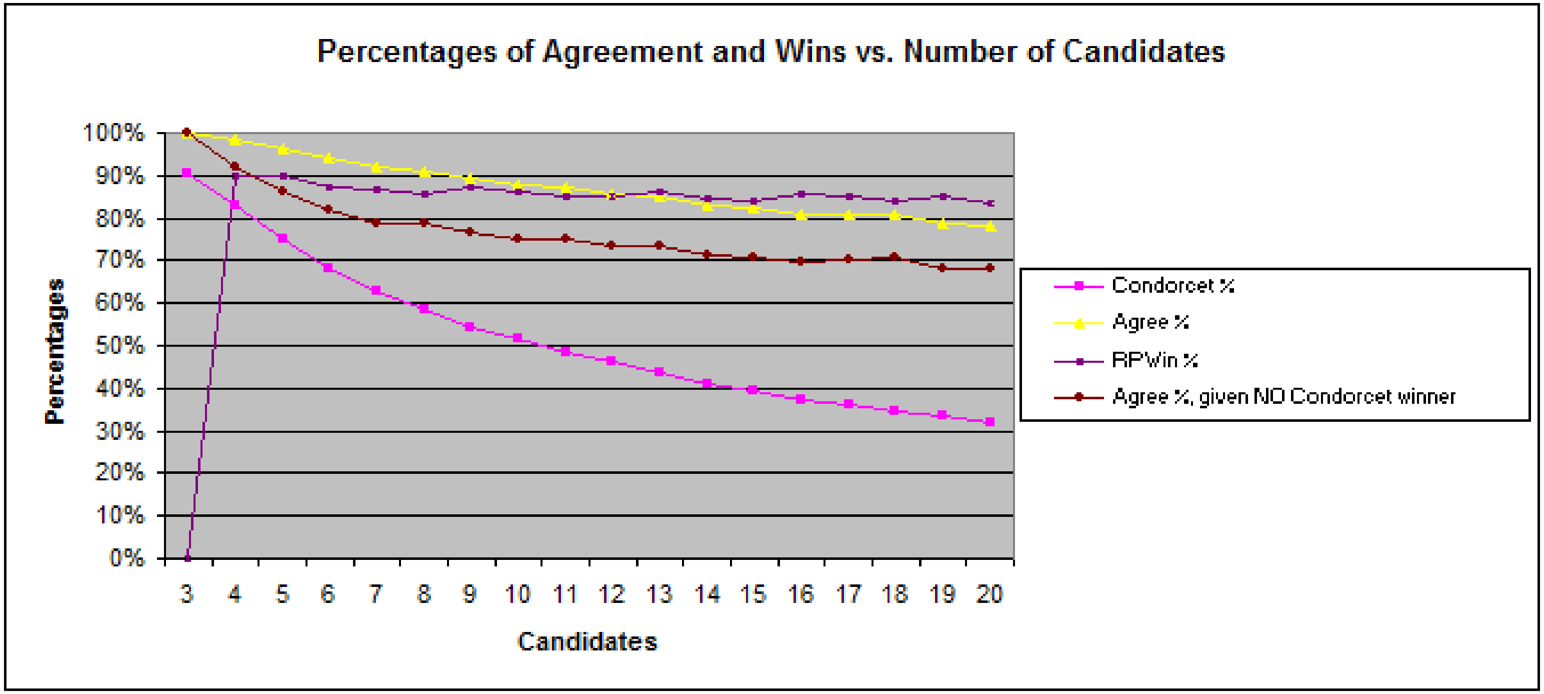
- N>3 Candidate Case: 1,000 voters, 10,000 sample elections

Number of Candidates	Agreement %	LWD Win %	Borda Win %
3	88.56%	100.00%	0.00%
4	81.66%	89.20%	10.80%
5	78.74%	84.95%	15.05%
6	75.35%	81.74%	18.26%
7	73.59%	77.36%	22.64%
8	71.89%	75.45%	24.55%
9	69.65%	73.25%	26.75%
10	68.45%	72.04%	27.96%

Number of Candidates	Agreement %	RP Win %	Borda Win %
3	88.56%	100.00%	0.00%
4	81.49%	92.76%	7.24%
5	78.15%	90.94%	9.06%
6	74.87%	89.38%	10.62%
7	72.13%	86.40%	13.60%
8	70.75%	85.95%	14.05%
9	68.19%	84.19%	15.81%
10	66.84%	85.07%	14.93%

LWD vs. Ranked Pairs

- N>3 Candidate Case: 10,000 voters, 10,000 sample elections



LWD vs. Ranked Pairs

- N>3 Candidate Case: 10,000 voters, 10,000 sample elections

Number of Candidates	RPwin %	Agree%	Condorcet%	% Agree given NO Condorcet
3	N/A	100.00%	90.57%	100.00%
4	90.00%	98.60%	82.80%	91.86%
5	89.88%	96.54%	75.10%	86.10%
6	87.33%	94.16%	68.01%	81.74%
7	86.93%	92.12%	62.90%	78.76%
8	85.76%	91.22%	58.28%	78.95%
9	87.19%	89.23%	54.33%	76.42%
10	86.33%	87.93%	51.37%	75.18%
11	85.29%	87.08%	48.40%	74.96%
12	84.98%	85.62%	46.05%	73.35%
13	85.98%	85.09%	43.79%	73.47%
14	84.54%	83.05%	40.95%	71.30%
15	83.99%	82.32%	39.28%	70.88%
16	85.53%	80.99%	37.08%	69.79%
17	85.18%	80.90%	36.07%	70.12%
18	84.03%	80.90%	34.39%	70.89%
19	85.04%	78.75%	33.50%	68.05%
20	83.64%	78.24%	31.85%	68.07%

Conclusion

- Voting system must satisfy major properties AND be practical and feasible to implement
- Ranked Pairs is best in terms of properties and head-to-head winner performance
  - Agreement with LWD high for n<10 candidates
  - Not as easy to program and understand
- LWD may be better for large candidate and voter pools in terms of run time

Suggestions for Future Research

- What are the best performing methods given certain circumstances?
  - Condorcet winner, lack of simple majority, etc.
- How do more complex systems compare?
  - E.g. Schulze, Kemeny-Young
- How do systems fare with other properties?
  - E.g. Clone Invariance
- Consider other models for voter behavior
  - Not purely random elections
  - Different distributions on unit interval model